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Entanglement entropy and its imprint in DIS hadron multiplicity distributions

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Entanglement entropy has emerged as a novel tool for probing QCD phenomena. In this talk I present recent results in describing hadron production in Deep Inelastic Scattering (DIS) of electrons on protons. In particular I will discuss recent results on the observation of entanglement entropy in a finite rapidity window, as measured by the H1 collaboration.

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1. Introduction

Entanglement is a direct consequence of the quantum mechanical description of nature. It has been first predicted and then observed and verified at both microscopic and macroscopic scales. While entanglement has been first verified within the realm of atomic physics, there is no doubt that it is also omnipresent at the femto-scale. Indeed, recently both ATLAS and CMS collaborations reported the observation of spin entanglement in a pair of top quarks, produced in proton proton collisions at the Large Hadron Collider [1–3]. While confirmation of entanglement in proton proton collisions at high energy is of interest itself, it is a natural question to ask what entanglement can teach us about the proton and the dynamics of the strong nuclear force. With Quantum Chromodynamics (QCD) established as the correct microscopic description of the dynamics of strongly interacting matter for more than 50 years, a precise understanding of the proton wave function and the mechanism which governs confinement of quarks and gluons into hadrons is still to be achieved. It has been argued [4, 5] that the phenomena of color confinement can be interpreted as realization of maximal entanglement: quarks and gluons inside a hadron are entangled to a degree that it is even in principle not possible to isolate and study them as separate entities.

A measure of the degree of entanglement of a pure quantum state is provided by entanglement entropy. With $\{|i_A\rangle\}$, $\{|j_B\rangle\}$ orthonormal sets of states in Hilbert spaces $\mathcal{H}_A$ and $\mathcal{H}_B$, a pure but entangled space in the product Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$ can be written as

$$|\psi\rangle = \sum_{i,j} c_{ij} |i_A\rangle |j_B\rangle, \tag{1}$$

with $c_{ij} \neq a_i b_j$ i.e. the state cannot be factorized into a product state. Using the singular value decomposition, such an entangled state can be rewritten in the form

$$|\psi\rangle = \sum_{k} \lambda_k |k_A\rangle |k_B\rangle, \tag{2}$$

where $|k_A\rangle, |k_B\rangle$ are orthonormal states in $\mathcal{H}_A$, $\mathcal{H}_B$, called the Schmidt basis; it differs in general from the original basis $\{|i_A\rangle\}$, $\{|j_B\rangle\}$. If in an experiment we only observe elements of the Hilbert space $\mathcal{H}_A$, i.e. all relevant operators are of the form $\hat{O} = \hat{O}_A \otimes \hat{\mathbb{1}}_B$, it is natural to determine the reduced density operator for the Hilbert space $\mathcal{H}_A$,

$$\rho_A = \mathrm{tr}_B |\psi\rangle\langle\psi| = \sum_k |\lambda_k|^2 |k_A\rangle\langle k_A|. \tag{3}$$

With $p_k = |\lambda_k|^2$, this is the density matrix of a mixed state in the Hilbert space $\mathcal{H}_A$, if the original state $|\psi\rangle$ is an entangled state. The von-Neumann entropy in $\mathcal{H}_A$,

$$S = -\mathrm{tr}_A \left(\rho_A \ln \rho_A\right) = -\sum_k p_k \ln p_k, \tag{4}$$

is then a measure of the degree of entanglement of the state $|\psi\rangle$. If the dimension d of the Hilbert space $\mathcal{H}_A$ is finite, we have moreover the upper bound $S \leq \ln d$, which is fulfilled for the homogeneous distribution $p_k = 1/d$. In [6] it has then been argued that such a scenario is realized in Deep Inelastic Scattering of electrons on protons (DIS). The virtual photon with virtually Q^2

exchanged between proton and electron resolves only certain parts of the proton wave function. Indeed, the condition $Q^2 \gg \Lambda^2_{\text{QCD}}$, with Λ_{QCD} the QCD characteristic scale of the order of a few hundred MeV, is essential for resolving in the DIS reaction quark degrees of freedom inside the proton.

To test these ideas one needs in principle full control over the proton wave function as well as a precise understanding of the unobserved Hilbert space, which at the moment is not at our disposal. As an intermediate solution to this problem it appears therefore to be meaningful to explore the proposal in regions of phase space where approximate expressions for the proton wave function are at our disposal and where natural degrees of freedom provide at the very least a good approximation to the Schmidt basis. An example of such a kinematic limit is provided by the low x limit of DIS at high photon virtuality Q^2. In this limit the invariant mass W of the produced hadronic system is large, $W^2 = (1-x)Q^2/x + m_p^2$, with m_p the proton mass. As a consequence the produced entropy can be expected to be sizeable in this limit and a comparison with experimental data is possible.

2. Entanglement entropy from the color dipole model

A suitable way to describe multiple production of gluons at low x is provided by the color dipole formalism [7], which identifies color dipoles as the natural degrees of freedom to describe DIS at low x; the color dipole Fock basis is therefore a natural candidate for the Schmidt basis in this kinematic limit. Even though it is possible to formulate an evolution equation for the probability density to encounter n color dipoles at transverse positions $\boldsymbol{r}_1, \ldots, \boldsymbol{r}_n$, the solution of the resulting system of equations is complicated and plagued by the emergence of large color dipoles sizes [8]. It is therefore meaningful to turn for a first exploration to a one-dimensional model of the low x evolution of color dipoles for the probability p_n to encounter n dipoles, which reads

$$\frac{dp_n}{dY} = -n\Delta p_n + (n-1)\Delta p_{n-1}. \tag{5}$$

It is solved by

$$p_n(Y) = \frac{e^{-\Delta Y}}{C}\left(1 - \frac{e^{-\Delta Y}}{C}\right)^{n-1}. \tag{6}$$

Here $Y = \ln 1/x$ denotes the total available phase space in rapidity, while $\Delta \simeq 0.2 - 0.4$ is the BFKL intercept within the one dimensional dipole model. The choice $C = 1$ is special, since it fixes the initial condition $p_1(0) = 1$, $p_{n\geq 2} = 0$, i.e. the system is in a pure state at $Y = 0$. The mean number of dipoles is obtained as $\bar{n}(Y) = Ce^{\Delta Y}$. To compare the results provided by this model to experiment, we make use of the hadronic entropy extracted by the H1 collaboration in [10] , which determined entropy from the multiplicity of the produced hadrons at the HERA collider. When comparing to experimental data, it is important to keep in mind that only charged particles (predominantly charged pions) are observed and that it is needed to rescale the mean number and therefore C by a factor $2/3$, see also the discussion in [9]. The H1 collaboration provides two alternatives for the extracted hadronic entropy. In the first version, a fixed rapidity window of size $\Delta^*\eta = 4$ is used, whereas the second version employs a moving rapidity window of size $\Delta\eta = 1.4$, centered around the leading scattered quark in the DIS reaction, see [10, 11] for details.

Whereas the restriction in phase space is a small correction in the first case (i.e. a good description can be obtained within an inclusive setup, see [9, 12]), the moving rapidity window of size $\Delta\eta = 1.4$ requires an extension of the inclusive framework. Such an extension has been proposed in [11]. Counting emissions only in the rapidity interval $[Y_0, Y]$ and not in the region $[0, Y_0]$, one has for the probability to have no emission into the interval $[Y_0, Y]$,

$$\tilde{p}_0(Y, Y_0) = C_0 e^{-\Delta(Y - Y_0)}, \tag{7}$$

where the dipole model sets $C_0 = 1$, while successful phenomenology requires a value $C_0 \in [0.655, .777]$. For the probabilities to have emission into the interval $[Y_0, Y]$, one has on the other hand,

$$\tilde{p}_{n\geq 1}(Y, Y_0) = [1 - \tilde{p}_0(Y, Y_0)] \cdot p_n(Y), \tag{8}$$

with $p_n(Y)$ the above inclusive probabilities. Entanglement entropy restricted to a certain rapidity window is then determined as

$$S_{\text{local}} = -\sum_n \tilde{p}_n \ln \tilde{p}_n = -\tilde{p}_0 \ln \tilde{p}_0 - (1 - \tilde{p}_0)\ln(1 - \tilde{p}_0) + (1 - \tilde{p}_0) S_{\text{inc.}}, \tag{9}$$

with $S_{\text{inc.}}$ the inclusive entanglement entropy. In the following we present results for which the

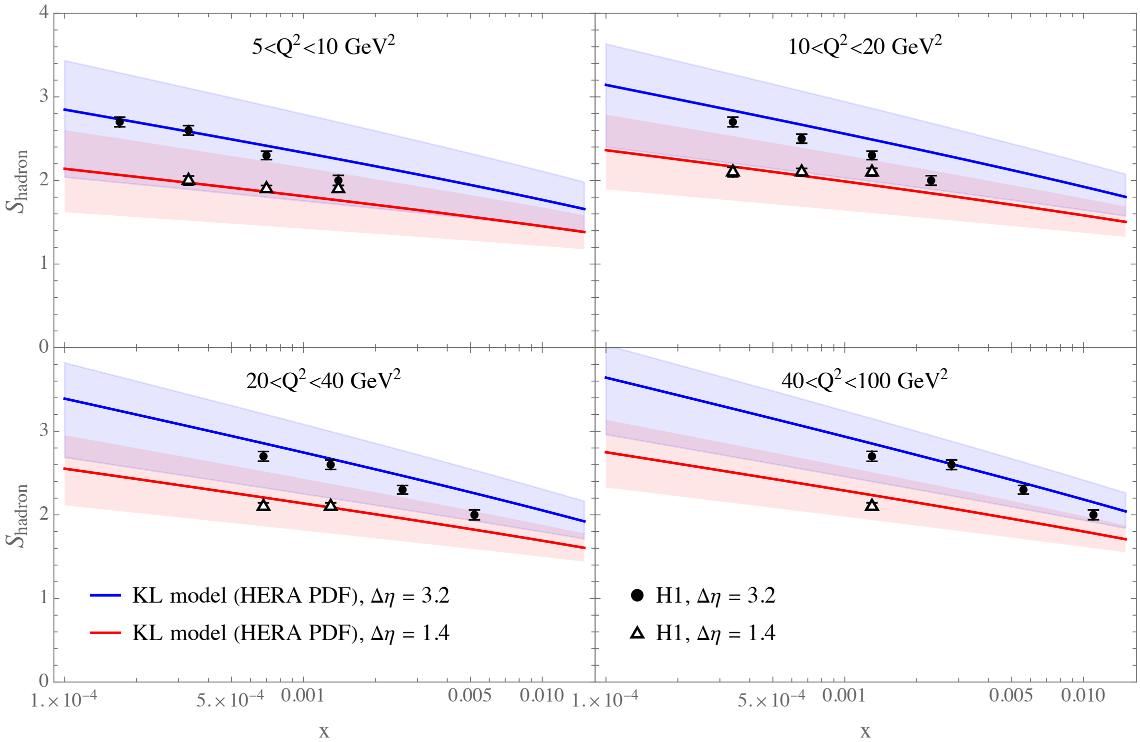

Figure 1: Hadronic entropy as obtained from Eq. (9).

inclusive entanglement entropy has been determined from Eq. (6), with $C = 1$ (but with the aforementioned rescaling by a factor $2/3$), while Δ is extracted from a fit to the x dependence of

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leading order HERAPDFs [13], see [11] for details. The constant $C_0 = 0.655 \pm 0.009$ was on the other hand determined from a simultaneous fit to both data in the narrow detection window, centered around the scattered quarks with $\Delta\eta = 1.4$ and the fixed detection window in the current fragmentation region with $\Delta\eta = 3.2$, corresponding to a pseudorapidity window in the hadronic center of mass frame of four units. Our numerical results, including a comparison to data are presented in Fig. 1. We find overall a very good agreement with the measured data set.

Apart from above results, which have been obtained within the inclusive DIS process, entanglement entropy has been furthermore studied for diffractive DIS, see [5]. For related studies which explore entanglement entropy in the context of strong interactions and in particular for DIS see also [14–24] for additional explorations. While at the current stage it cannot be excluded with certainty that the agreement with data is a coincidence, evidence for the observation of entanglement entropy in DIS data is growing.

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