PROCEEDINGS OF SCIENCE

Quantum Entanglement in Particle-Antiparticle Pair Production under Time-Dependent Electromagnetic Fields

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Quantum entanglement, a defining feature of quantum theory, is generated dynamically by the Schwinger effect—the production of particle-antiparticle pairs from a strong electric field. In this work, we go beyond traditional particle-number analyses to study the quantum entanglement generated in this process. We partition the system into momentum modes and calculate entanglement entropy and logarithmic negativity as functions of time. For a single Sauter pulse, entanglement exhibits a monotonic growth that closely tracks the pair creation process. Strikingly, a train of alternating pulses induces coherent interference, leading to a cumulative enhancement of entanglement and revealing a clear interplay between successive creation events. Most significantly, we show that the temporal delay (T) between pulses acts as a tunable parameter, controlling the level of entanglement via constructive or destructive interference. This tunability provides a distinct signature of vacuum memory, establishing pair production as a platform for entanglement-controlled strong-field physics.

QCD at the Extremes (QCDEX2025)
1–5 Sept 2025
Online

[*]Speaker

ISSN 1824-8039. Published by SISSA Medialab.

PoS(QCDEX2025)033

1. Introduction

Quantum entanglement stands as one of the most profound and distinctive features of quantum theory, describing non-local correlations between systems that have no classical counterpart. Beyond its foundational significance, entanglement is the core resource enabling quantum technologies—such as quantum computation, cryptography, and teleportation—to achieve capabilities beyond classical limits [1, 2]. A key challenge in harnessing this resource is the controlled generation of entangled states. One of the most striking physical mechanisms for generating entanglement from the vacuum is the Schwinger effect: the production of particle–antiparticle pairs from a strong electric field.

The Schwinger effect, a cornerstone of strong-field QED, was first predicted by Sauter [4] and later formalized by Schwinger[3]. It arises when an external electric field imparts sufficient energy to virtual particle–antiparticle fluctuations in the quantum electrodynamic (QED) vacuum, promoting them into real observable pairs. While the traditional Schwinger limit requires extremely high field strengths, recent advances suggest that pair production can be significantly enhanced in time-dependent fields, such as those produced by ultra-intense lasers or engineered pulse sequences. This has renewed theoretical and experimental interest in the effect [5]. The spectrum of particles produced in time-dependent electric fields has been extensively studied using techniques including quantum kinetic theory [6–8], the Dirac–Heisenberg–Wigner formalism [9], and scattering approaches [10]. A central insight is that quantum interference plays a crucial role in the production process. Semiclassical methods have been used to design electric field profiles that enhance or suppress particle yields, demonstrating a degree of quantum control over pair creation[11].

More recently, attention has turned to the quantum state of the produced pairs. It is known that particles and antiparticles of a given momentum mode are generated in an entangled state—a two-mode squeezed state—due to the coherence of the pair production process [12, 13]. However, the dynamical evolution of this entanglement, especially under complex field configurations, remains less explored. In this work, we focus on quantifying and controlling this entanglement using tools from quantum information theory, specifically the entanglement entropy and the logarithmic negativity. The latter provides a robust measure of distillable entanglement, even for mixed states, and allows us to track the genuinely quantum correlations between particles and antiparticles as the field evolves [14].

The phenomenon of entanglement generation in pair production is not unique to QED. Analogous processes appear in diverse systems, including atomic ionization [15], Landau–Zener transitions [18], cosmological particle production [17], Hawking radiation [16]. Insights gained from the Schwinger effect can thus inform studies across physics, from condensed matter to gravity.

In this paper, we investigate the generation and evolution of quantum entanglement in spinor QED in (1+1) dimensions under time-dependent electric fields. Using Sauter-type pulses as a model, we consider both single- and multi-pulse configurations within the canonical quantization framework. We construct the time-dependent entangled state of particle–antiparticle pairs and quantify entanglement through entropy and logarithmic negativity.

Our analysis reveals how a single pulse generates entanglement that monotonically tracks pair creation, and how a pulse train produces interference effects, with the temporal delay T between pulses acting as a tunable control parameter. The paper is organized as follows: Section 2 outlines the theoretical framework, including the quantization of the Dirac field, the Bogoliubov transformation, and the formulation of the two-mode squeezed state, along with the entanglement measures employed. Section 3 analyzes the single-pulse case, followed by the extension to pulse trains, where interference and timing effects are discussed. Conclusions are presented in Section 4.

Throughout the paper, we use natural units and set $\hbar = c = m = |e| = 1$, and express all variables in terms of the electron mass unit.

2. Theory

We consider massive Dirac fermions in 1 + 1 dimensions coupled to a homogeneous, time-dependent electric field $E(t) = -\partial_t A_1(t)$. The action is

$$S = \int \mathrm{d}^2x\, \bar{\hat{\psi}} \left(i\gamma^\mu \partial_\mu - eA_1(t)\gamma^1 - m\right) \hat{\psi}. \tag{1}$$

The corresponding Hamiltonian density is:

$$\hat{\mathcal{H}}(t) = \hat{\psi}^\dagger \left[-i\gamma^0\gamma^1\partial_1 + e\gamma^0\gamma^1 A_1(t) + m\gamma^0\right] \hat{\psi}. \tag{2}$$

To analyze particle creation, we use the concept of an *instantaneous vacuum* [20]. At any finite time t, we solve the Dirac equation to find a complete set of spinor eigenmodes, $U_k(t)$ and $V_k(t)$.

Therefore, the quantum field operator is expanded in this instantaneous basis:

$$\hat{\psi}(t, x_1) = \int \frac{dk}{2\pi} \left[U_k(t)\hat{a}_k(t) + V_{-k}(t)\hat{b}^\dagger_{-k}(t)\right] e^{ikx_1}. \tag{3}$$

Here, $\hat{a}_k(t)$ and $\hat{b}_k(t)$ are the time-dependent particle and antiparticle annihilation operators defined at time t. Substituting this expansion into the Hamiltonian leads to its diagonal form:

$$\hat{H}(t) = \int \frac{dk}{2\pi}\, \omega_k(t) \left[\hat{a}^\dagger_k(t)\hat{a}_k(t) + \hat{b}^\dagger_{-k}(t)\hat{b}_{-k}(t)\right], \tag{4}$$

where $\omega_k(t) = \sqrt{m^2 + (k - eA(t))^2}$ is the instantaneous energy. This defines the instantaneous vacuum $|\Omega_t\rangle$ via $\hat{a}_k(t)|\Omega_t\rangle = \hat{b}_k(t)|\Omega_t\rangle = 0$.

Particle creation arises because the vacuum state is not invariant under time evolution.

The relation between the asymptotic operators ($t \to -\infty$) and those at time t is given by a Bogoliubov transformation:

$$\hat{a}_k(t) = \alpha_k(t)\hat{a}_k(-\infty) + \beta^*_k(t)\hat{b}^\dagger_{-k}(-\infty), \tag{5}$$

$$\hat{b}_{-k}(t) = \alpha_k(t)\hat{b}_{-k}(-\infty) - \beta^*_k(t)\hat{a}^\dagger_k(-\infty), \tag{6}$$

with the normalization $|\alpha_k(t)|^2 + |\beta_k(t)|^2 = 1$. The coefficient $|\beta_k(t)|^2$ is the mean number of pairs created in mode k by time t.

To compute these coefficients numerically, we employ the method of Ref. [19], which avoids rapidly oscillating phases. We define:

$$c^{(1)}_k(t) = \alpha_k(t)e^{-i\int^t dt'\,\omega_k(t')}, \tag{7}$$

$$c^{(2)}_k(t) = \beta_k(t)e^{+i\int^t dt'\,\omega_k(t')}. \tag{8}$$

These satisfy the differential system:

$$i\frac{d}{dt}\begin{bmatrix} c_k^{(1)}(t) \\ c_k^{(2)}(t) \end{bmatrix} = \begin{pmatrix} \omega_k(t) & i\Lambda_k(t) \\ -i\Lambda_k(t) & -\omega_k(t) \end{pmatrix} \begin{bmatrix} c_k^{(1)}(t) \\ c_k^{(2)}(t) \end{bmatrix}, \tag{9}$$

where $\Lambda_k(t) = \frac{eE(t)m}{2\omega_k^2(t)}$. The Bogoliubov coefficients are then given by $|\alpha_k(t)|^2 = |c_k^{(1)}(t)|^2$ and $|\beta_k(t)|^2 = |c_k^{(2)}(t)|^2$.

2.1 Time-Dependent Quantum State and Entanglement Measures

The time-evolved initial vacuum is an entangled state for each momentum pair $(k, -k)$. It is a two-mode squeezed state [13]:

$$|\psi_k(t)\rangle = \alpha_k^*(t)|0_k, 0_{-k}\rangle + \beta_k^*(t)|1_k, 1_{-k}\rangle, \tag{10}$$

where the kets represent the particle and antiparticle Fock states.

For a more robust entanglement quantification that works for mixed states, we compute the logarithmic negativity. For the bipartite state $\rho_{k,-k}(t) = |\psi_k(t)\rangle\langle\psi_k(t)|$, the partial transpose with respect to one subsystem has eigenvalues:

$$\lambda_{1,2} = |\alpha_k(t)|^2, \quad \lambda_{3,4} = \pm|\alpha_k(t)\beta_k(t)|. \tag{11}$$

For our state, logarithmic negativity is given by:

$$\mathcal{N}_{log}(t) = \int \frac{dk}{(2\pi)} \log_2 \left(1 + 2|\alpha_k(t)\beta_k(t)|\right). \tag{12}$$

Tracing out the antiparticle degree of freedom ($-k$ mode) yields the reduced density matrix for the particle mode [13]:

$$\rho_k(t) = \mathrm{Tr}_{-k}(|\psi_k(t)\rangle\langle\psi_k(t)|) = \begin{pmatrix} |\alpha_k(t)|^2 & 0 \\ 0 & |\beta_k(t)|^2 \end{pmatrix}. \tag{13}$$

The entanglement entropy is its von Neumann entropy [12]:

$$S_k(t) = -|\alpha_k(t)|^2 \log_2 |\alpha_k(t)|^2 - |\beta_k(t)|^2 \log_2 |\beta_k(t)|^2. \tag{14}$$

The total entropy is

$$S(t) = \int \frac{dk}{2\pi} S_k(t) \tag{15}$$

2.2 Field Configuration

We investigate a train of N alternating-sign Sauter pulses:

$$E(t) = E_0 \sum_{j=1}^{N} (-1)^{j-1} \operatorname{sech}^2\left(\frac{t - t_j}{\tau}\right), \tag{16}$$

where $t_j = \left(j - \frac{N+1}{2}\right)T$ is the center of the j-th pulse, E_0 is the peak field strength, τ is the pulse duration, and T is the temporal delay between pulses. This configuration allows us to study interference and memory effects in entanglement generation.

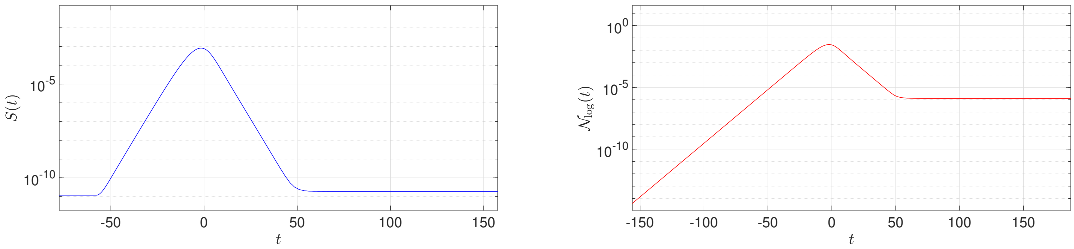

Figure 1: Left panel :Evolution of the entanglement entropy, $S(t)$ for single pulse field with parameters $E_0 = 0.1E_c, \tau = 80[m^{-1}]$. Right panel :Logarithmic negativity $\mathcal{N}_{\log}$ under a single-pulse field with parameters $E_0 = 0.1E_c$ and $\tau = 20[m^{-1}]$

3. Results

All results are obtained by numerically solving the coupled differential equations (9)for Sauter-type pulsed field with a field strength of $E_0 = 0.1E_c$, where the critical Schwinger field is $E_c = \frac{m^2}{|e|}$.All quantities are expressed in natural units based on the electron mass $[m]$.

To establish a baseline, we first analyze the generation of quantum entanglement under a single time-dependent electric pulse. The dynamics are quantified using the two complementary measures: the entanglement entropy $S(t)$ and the logarithmic negativity $\mathcal{N}_{\log}(t)$.

The results for a single Sauter pulse with electric field parameters $E_0 = 0.1E_c$ and $\tau = 20[m^{-1}]$ are presented in Figure 1. The left panel shows the entanglement entropy $S(t)$, which measures the total quantum correlations between the particle-antiparticle pairs. It rises from zero as the electric field pulse becomes significant, peaks during the pulse's maximum, and saturates to a constant after the pulse has passed. This saturation confirms the permanent creation of entangled particle-antiparticle pairs. The final value of the entropy is directly related to the asymptotic particle production probability $|\beta_k(\infty)|^2$.

The right Panel of Figure 1 displays the logarithmic negativity $\mathcal{N}_{\log}(t)$, a measure dedicated to quantifying purely quantum entanglement. It displays a nearly identical temporal profile, growing in tandem with the entropy during the pulse and also plateauing to a finite value at late times. The close tracking of $\mathcal{N}_{\log}(t)$ with $S(t)$ confirms that the correlations generated are predominantly quantum-mechanical in nature. The single-pulse case demonstrates that the electric field acts as a source of quantum entanglement, with the particle and antiparticle of a produced pair created in a highly non-classical, entangled state.

We now extend the analysis to a more complex field configuration: a train of four electric-field pulses with alternating directions. The results for this field are shown in Figure 2. Unlike the single, smooth rise and plateau observed for a single pulse, bot the entanglement entropy (left panel) and logarithmic negativity (right panel) now exhibit a series of rapid oscillations and sharp peaks. Each new pulse in the train interacts with the existing quantum state, causing a jump and complex oscillatory structure in the entanglement. Critically, the overall entanglement level increases cumulatively with each successive pulse, building up the total quantum correlations. The fact that both $S(t)$ and $\mathcal{N}_{log}(t)$ follow this identical complex pattern confirms that the particle-antiparticle pairs created in the multi-pulse scenario remain in a genuinely quantum-entangled state. This demonstrates that a pulse train does not merely create independent entangled pairs but

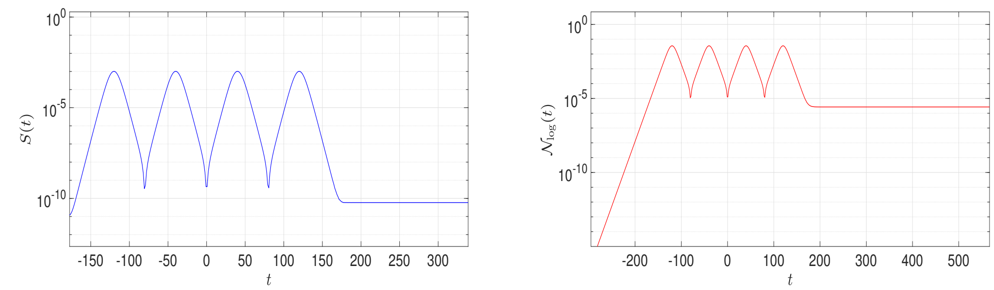

Figure 2: Time evolution of entanglement entropy (left) and logarithmic negativity (right) under a four-pulse electric field train with parameters $E_0 = 0.1E_c, \tau = 20[m^{-1}], T = 80[m^{-1}]$.

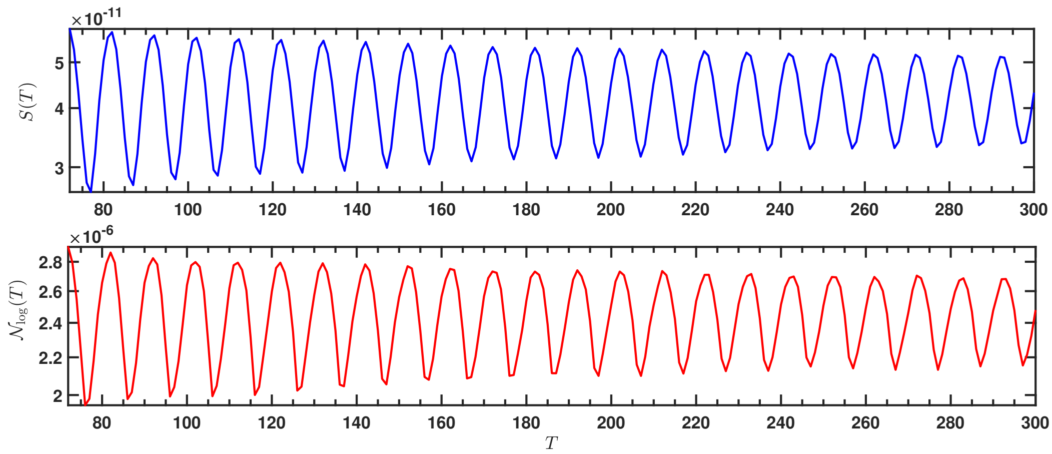

Figure 3: Entanglement entropy S (top) and logarithmic negativity $\mathcal{N}_{\log}$ (bottom) vs. temporal delay T.

induces a coherent interaction between successive pair creation events, leading to a richer and more intense entanglement structure.

Figure 3 presents the asymptotic entanglement, quantified by both the entanglement entropy (top) and the logarithmic negativity (bottom), as a function of the temporal delay T between pulses. The most salient feature is a pronounced oscillatory dependence on T, manifesting as a regular pattern of peaks and valleys. The oscillations in both measures are perfectly in phase, confirming this is a robust feature of the quantum entanglement itself. The peaks correspond to delays where the pair production amplitudes from successive pulses interfere constructively, thereby enhancing the final entanglement. Conversely, the valleys correspond to delays that induce destructive interference, suppressing the entanglement yield. The slight decay in the oscillation amplitude with increasing T suggests a gradual decoherence effect at larger separations.
This coherent interference pattern demonstrates that the time delay T acts as a direct control parameter for tuning the non-local correlations in the produced particle-antiparticle pairs.

4. Conclusion

In this work, we have quantitatively analyzed the generation and control of quantum entanglement in the non-perturbative Schwinger mechanism. By employing quantum information measures-the entanglement entropy and the logarithmic negativity - we have characterized the quantum state of particle-antiparticle pairs produced in time-dependent electric fields. In summary, our study establishes that: 1. A single Sauter pulse generates quantum entanglement that grows monotonically, directly mirroring the temporal profile of the pair creation process itself. 2. A train of alternating pulses induces coherent interference between successive pair creation events, leading to a cumulative and oscillatory enhancement of entanglement far beyond the single-pulse case. 3. The temporal delay (T) between pulses acts as a powerful and tunable parameter, controlling the level of entanglement through constructive and destructive interference. This oscillatory dependence provides a direct signature of vacuum memory, indicating that the quantum vacuum retains a phase coherence over time.

These findings demonstrate a rich interplay between quantum coherence, strong-field dynamics, and non-perturbative vacuum processes. More broadly, they establish pair production in strong fields as a dynamic platform for generating and controlling non-local quantum correlations. The ability to manipulate entanglement through classical field parameters like pulse timing opens up novel pathways for designing quantum experiments in high-energy physics and suggests potential applications in developing schemes for quantum information processing in relativistic regimes.

References

[1] P. M. Alsing, I. Fuentes-Schuller, R. B. Mann and T. E. Tessier, Phys. Rev. A **74**, 032326 (2006) doi:10.1103/PhysRevA.74.032326 [arXiv:quant-ph/0603269 [quant-ph]].

[2] Y. Shi, Phys. Rev. D **70**, 105001 (2004) doi:10.1103/PhysRevD.70.105001 [arXiv:hep-th/0408062 [hep-th]].

[3] J. S. Schwinger, Phys. Rev. **82**, 664-679 (1951) doi:10.1103/PhysRev.82.664

[4] F. Sauter, Z. Phys. **69**, 742-764 (1931) doi:10.1007/BF01339461

[5] A. Fedotov, A. Ilderton, F. Karbstein, B. King, D. Seipt, H. Taya and G. Torgrimsson, Phys. Rept. **1010**, 1-138 (2023) doi:10.1016/j.physrep.2023.01.003 [arXiv:2203.00019 [hep-ph]].

[6] D. Sah and M. P. Singh, [arXiv:2301.06545 [hep-ph]].

[7] S. M. Schmidt, D. Blaschke, G. Ropke, S. A. Smolyansky, A. V. Prozorkevich and V. D. Toneev, Int. J. Mod. Phys. E **7**, 709-722 (1998) doi:10.1142/S0218301398000403 [arXiv:hep-ph/9809227 [hep-ph]].

[8] D. Sah and M. P. Singh, Acta Phys. Polon. A **148**, no.2, 149 (2025) doi:10.12693/aphyspola.148.149

[9] F. Hebenstreit, R. Alkofer and H. Gies, Phys. Rev. Lett. **107**, 180403 (2011) doi:10.1103/PhysRevLett.107.180403 [arXiv:1106.6175 [hep-ph]].

[10] C. K. Dumlu, Phys. Rev. D **79**, 065027 (2009) doi:10.1103/PhysRevD.79.065027 [arXiv:0901.2972 [hep-th]].

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[11] G. V. Dunne and T. Hall, Phys. Rev. D **58**, 105022 (1998) doi:10.1103/PhysRevD.58.105022 [arXiv:hep-th/9807031 [hep-th]].

[12] Z. Ebadi and B. Mirza, Annals Phys. **351**, 363-381 (2014) doi:10.1016/j.aop.2014.09.002 [arXiv:1410.3130 [quant-ph]].

[13] Y. Li, Y. Dai and Y. Shi, Phys. Rev. D **95**, no.3, 036006 (2017) doi:10.1103/PhysRevD.95.036006 [arXiv:1612.01716 [hep-th]].

[14] Y. Li, Q. Mao and Y. Shi, Phys. Rev. A **99**, no.3, 032340 (2019) doi:10.1103/PhysRevA.99.032340 [arXiv:1812.08534 [hep-th]].

[15] F. Krausz and M. Ivanov, Rev. Mod. Phys. **81**, 163-234 (2009) doi:10.1103/RevModPhys.81.163

[16] M. K. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042-5045 (2000) doi:10.1103/PhysRevLett.85.5042 [arXiv:hep-th/9907001 [hep-th]].

[17] P. Semrén, Phys. Rev. D **110**, no.8, 085004 (2024) doi:10.1103/PhysRevD.110.085004 [arXiv:2406.18205 [gr-qc]].

[18] S. N. Shevchenko, S. Ashhab and F. Nori, Phys. Rept. **492**, 1 (2010) doi:10.1016/j.physrep.2010.03.002 [arXiv:0911.1917 [cond-mat.supr-con]].

[19] E. Akkermans and G. V. Dunne, Phys. Rev. Lett. **108**, 030401 (2012) doi:10.1103/PhysRevLett.108.030401 [arXiv:1109.3489 [hep-th]].

[20] D. Sah and M. P. Singh, [arXiv:2309.12079 [hep-ph]].