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Status of muon g-2 theory

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Almost twenty years ago, physicists at Brookhaven National Laboratory measured the muon magnetic moment with a remarkable precision of 0.54 parts per million. Since then, a persistent discrepancy between experimental results and theoretical predictions has hinted at the tantalising possibility of undiscovered forces or elementary particles. However, results from lattice QCD now suggest a resolution to this longstanding discrepancy.

In this contribution, I review the current status of theoretical determinations of the muon anomalous magnetic moment in the context of the latest experimental results. I will particularly focus on our evolving understanding of the HVP, and what this means for the possibility of new physics in this longstanding discrepancy.

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1. Overview

For almost twenty years, the muon magnetic moment has presented a discrepancy that hints at new physics beyond the Standard Model of particle physics. Theoretical predictions based on the Standard Model had a strong disagreement with extremely precise experimental measurements [20]. As shown in Figure 1, the latest experimental result [2] shows almost six sigma tension with the 2020 theory consensus [9]. This should be sufficient evidence for a discovery, however in the meantime several new results have called in to question the accuracy of the 2020 theory prediction.

First, in 2021 the Budapest-Marseille-Wuppertal (BMW) collaboration published a significant new theoretical calculation [3] that seemed to resolve the tension with experiment. Bringing together cutting-edge numerical techniques and world-class supercomputing resources, this study provided

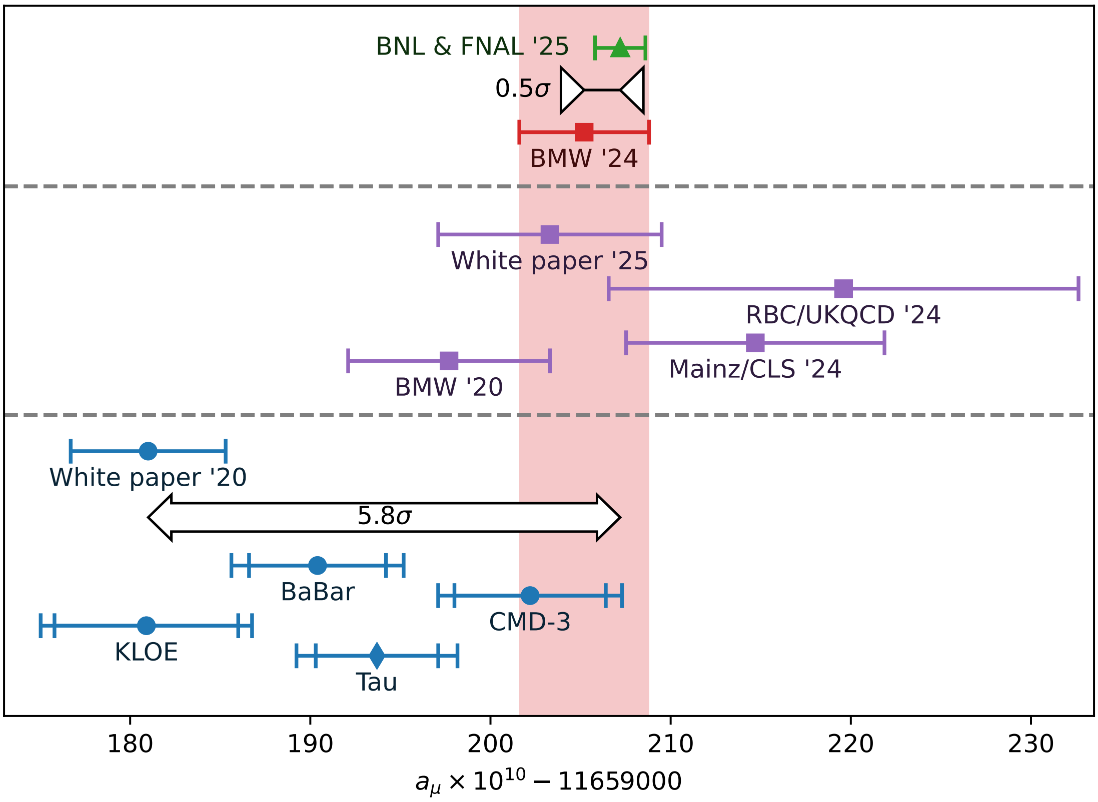

Figure 1: Comparison of standard-model predictions for the muon anomalous magnetic moment with its measured value, taken from Ref. [1]. The top panel shows a comparison of the world-average experimental measurement of a_μ [2] with the Standard Model prediction obtained by the BMW collaboration [1], denoted by the red band. The middle panel shows a predictions based on the earlier BMW result from Ref. [3] as well as results from two other similar calculations, from RBC/UKQCD [4–6] and Mainz [7], along with the consensus combination of the three from Ref. [8]. The lower panel shows the predictions for $a_\mu^{\text{LO-HVP}}$ obtained in the 2020 approach [9] using specific experimental inputs [10]. These correspond to BaBar [11, 12], KLOE [13–16] and CMD-3 [17], and τ decays [18, 19]. Note, all standard-model predictions include non-HVP contributions from “White paper ’25”, except for “White paper ’20”.

first-principles calculation of the main contribution of the strong force to the magnetic moment to an unprecedented level of precision. However, this result created a new tension, this time between two Standard Model determinations of this contribution, as seen in Figure 1. A recent update [1] improved the precision and brought the theory value even closer to the experiment.

Secondly, the 2020 theory consensus relied on key experimental inputs to determine the strong force contributions. In 2023, a new result from the CMD-3 collaboration [17] and a re-analysis of Tau decay data [18, 19] provided significantly different values for these inputs, eventually leading to the deprecation of this consensus in 2025 [8]. This situation is shown in the lower part of Figure 1.

Finally, in 2024, two other theory collaborations completed calculations that agreed with the BMW result, confirming that the standard model agreed remarkably well with experiment. In 2025, the theory consensus was updated to use a combination of these results [8], bringing it in line with the experimental result, albeit with a larger uncertainty than the 2024 BMW result.

2. Anomalous magnetic moment

The discrepancy specifically shows up in the so-called anomalous magnetic moment, defined in terms of the magnetic moment $\vec{\mu} = g\left(\frac{q}{2m}\right)\vec{S}$. The anomalous magnetic moment is the relative deviation of g from its tree-level value of 2. This anomalous magnetic moment is measured experimentally by injecting a polarised beam of muons into a large magnetic storage ring and observing the decay of the muons into positrons. As the muons cycle around the ring, their magnetic moments precess. The difference between the precession and orbital frequencies is directly related to the anomalous magnetic moment. It can be observed as a fluctuation in the energy distribution of the positrons as the momentum and magnetic moment come in and out of alignment.

The theoretical prediction is dominated by contributions from electromagnetism, which make up 99.994% of the total. These have been computed in perturbation theory up to five loops [21, 22]. The much smaller electroweak contributions have been computed to two loops [23, 24]. These provide extremely precise values for these parts of the total, as can be seen in Table 1.

The remaining contributions come from the strong force, and are: the hadronic vacuum polarisation (HVP), and the hadronic light by light scattering (HLbL). These effects contribute a very small amount to the total value, but they dominate the uncertainty, as seen in Table 1. These large uncertainties arise because we apply perturbation theory, and instead must use another approach. In the remainder of these proceedings we will focus on the HVP as it has the largest contribution to the uncertainty, and is where there have been significant theoretical developments.

Contribution	Value		
$a_\mu^{QED} \times 10^{10}$	11 658 471.88	±	0.02
$a_\mu^{EW} \times 10^{10}$	15.44	±	0.04
$a_\mu^{HVP} \times 10^{10}$	704.5	±	6.1
$a_\mu^{HLbL} \times 10^{10}$	11.55	±	0.99
$a_\mu \times 10^{10}$	11 659 203.3	±	6.2

Table 1: Individual contributions to the standard model theory consensus from Ref. [8].

3. Hadronic Vacuum Polarisation

The first approach, which was used to obtain the HVP in the 2020 consensus, is a data-driven evaluation [9, 25–28]. This approach utilises experimental measurements of hadron production at electron-positron colliders, which is related to the HVP using a dispersive approach.

At the time, known tensions between key experimental results from BaBar [11, 12] and KLOE [13–16], were considered in the uncertainty estimation. Since then, a new result from CMD-3 [17, 29] shows an even stronger tension, suggesting that this uncertainty was underestimated. This motivated a re-evaluation of experimental data, including revisiting data from tau decays [10]. The effects of each of these data sets on the final result can be seen in Figure 1. Individual results are spread across the whole range between the 2020 consensus and experiment, making their interpretation unclear.

Rather than using experimental data to determine the strong contributions, we can instead take the QCD+QED Lagrangian and perform a first-principles calculation. Since perturbation theory won't work, we instead turn to lattice QCD. By discretising spacetime and constraining the problem to a finite box, we render it finite and can perform Monte-Carlo sampling using supercomputers.

Until recently, lattice calculations had uncertainties significantly larger than data-driven determinations, and weren't able to provide insight into the discrepancy. The 2020 BMW result [3] was the first with uncertainties comparable to the data-driven approach. This result was in agreement with the experimental result, in contrast to the data-driven tensions. More recently, BMW produced a new, blinded result [1] with uncertainties even smaller than any of the data-driven results, and the Mainz [7] and RBC/UKQCD [4–6] collaborations produced their own independent results. As seen in Figure 1, all of these results largely agree with one another, and agree well with the experiment.

The 2020 BMW result was 3.4× more precise than our previous work [30], and the 2024 update was 1.7× more precise again. Figure 2 summarises the major uncertainties in the calculations, and

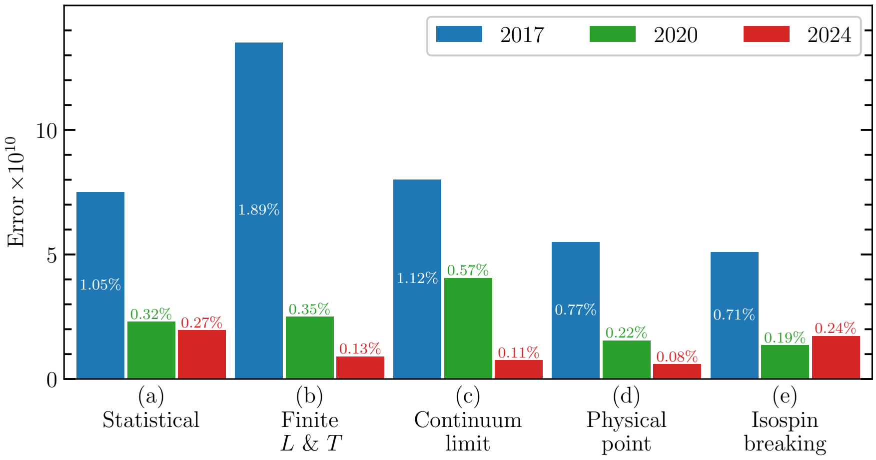

Figure 2: Dominant uncertainties in the BMW calculations of $a_\mu^{\text{LO-HVP}}$. There are five main sources, arising from: (a) statistical uncertainty from the Monte-Carlo sampling, (b) effects of the finite volume and time extent of the simulations, (c) extrapolating away the discretisation of the QCD Lagrangian, (d) physical inputs used to set the parameters of QCD, (e) isospin-breaking from QED and the up-down quark mass difference.

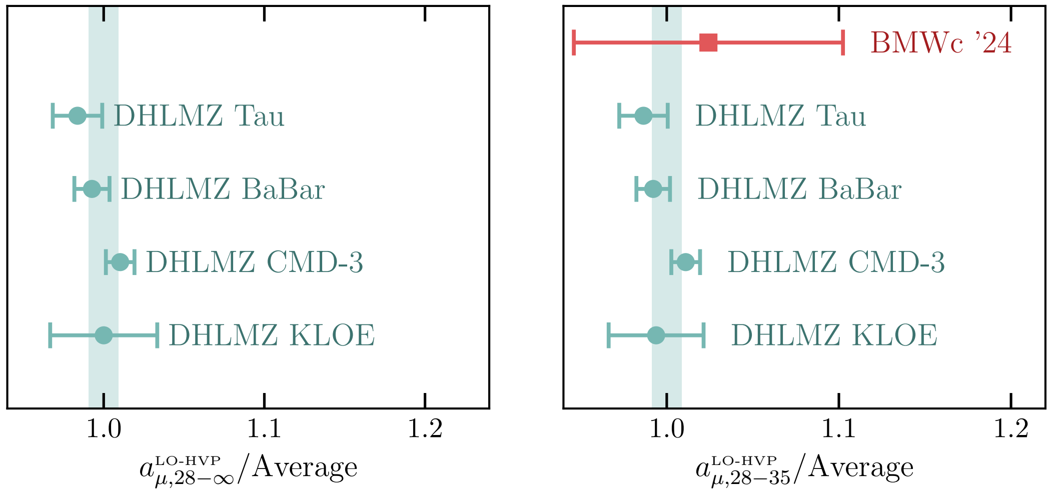

Figure 3: A comparison of relative deviations of data-driven results for the long-distance tail of the HVP with two-pion production data restricted to specific experimental data sets, including results from Tau decays [1]. The shaded bands represent an average of the data-driven points, and all points are relative to the central value of that average. The left panel is the entire tail from 2.8 fm to infinity, and the right panel is a comparison with lattice for the part of the tail from 2.8 fm to 3.5 fm. The scatter of the individual results is consistent with their uncertainties, and the lattice result agrees well, although its errors are larger.

shows the improvements that were needed to reach the current level of precision.

The dominant uncertainties in 2017 came from the effect of the finite physical size of our simulations. In 2020, this was addressed by performing dedicated simulations in a much larger volume. Once this uncertainty was reduced, the other uncertainties became significant and needed to be addressed, through algorithmic improvements [31, 32], new techniques for the exact treatment of infrared modes [33], new physical inputs, and direct calculations of all isospin-breaking effects.

In 2024, new simulations were performed with a finer discretisation, and the analysis procedure was overhauled. Finally, a novel approach was taken to address the challenging long-distance parts of the calculation. The long-distance tail beyond 2.8 fm has a large statistical uncertainty and finite volume effects, and a strong dependence on the physical inputs, despite only contributing about 5% of the total. However, this tail is dominated by low energies, well below the energy region where we observe discrepancies in the data-driven analysis. As a result, data-driven determinations of the tail give clear and consistent results, as shown in Figure 3. By replacing this small part of the total with a data-driven determination, the uncertainties are significantly reduced, allowing a result twice as precise as the pure-lattice consensus.

4. Conclusion

Recent calculations from the BMW collaboration have surpassed the precision of existing theoretical predictions of HVP contribution to the muon magnetic moment. When these results are combined with the latest theory consensus for the other contributions [8], we obtain a theory prediction that is in excellent agreement with experimental measurements [2]. This result is a remarkable success for renormalized quantum field theories, validating the Standard Model to 0.31 ppm.

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