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Theory overview on semileptonic decays

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Semileptonic decays play an essential role in the determination of the CKM matrix elements. We review the current status of theoretical calculation of semi-leptonic B decays, focusing on the method of light cone QCD sum rules. In this approach, people calculate the suitable correlation function with weak current between vacuum and B meson or light meson light cone distribution amplitudes. By using the dispersion relation and quark-hadron duality, we can get the corresponding transition form factors in the large recoil region. Recent progresses have been made up to the two loop QCD corrections and next-to-leading power corrections. These improvements combining with the lattice QCD results at small recoil region, have significantly improved the theoretical precision of $|V_{ub}|$ and $|V_{cb}|$ determination. We emphasize the importance of the non-perturbative input of meson light cone distribution amplitudes in these studies.

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1. Introduction

CP violation in the weak interaction originates from the quark mixing matrix. The unitarity test of this CKM matrix may give new physics signals beyond the standard model. Semileptonic weak decays play an essential role in the determination of the CKM matrix elements. Theoretically we can calculate the heavy quark semileptonic decays perturbatively, which is already in high precision. The heavy quark decay can be experimentally measured as heavy hadron inclusive decay if we assume quark-hadron duality. However, the precision of inclusive decay measurement is limited. On the other hand the exclusive decays such as $B \to D^{(*)}\ell\bar{\nu}_\ell$ or $B \to \pi(\rho)\ell\bar{\nu}_\ell$ are measured with very high precision in experiments. Theoretically, the decay rates of these decays depend on hadron transition form factors, which requires non-perturbative hadronization process. The precision of theoretical calculations for most exclusive heavy flavor processes need improvement.

Discrepancy exists between CKM matrix elements determination through inclusive and exclusive semi-leptonic decays, the so called V_{cb} and V_{ub} puzzle [1]. We need both more precise experimental measurement and higher order theoretical calculation to solve these puzzles. In addition, semileptonic decays are also sensitive to new physics signals, such as the (ever) existed anomalies $R(D^{(*)})$, $R(K^{(*)})$ [2] problem etc. We also need more precise theoretical calculation of form factors to solve these anomalies with more precise experimental data.

Recently, the lattice QCD has made a great progress in the determination of transition form factors for semi-leptonic B decays [3], which is valid only in the small recoil region. For the large recoil region, we have to rely on perturbative QCD calculations, such as light cone QCD sum rules (LCSR) with operator product expansion. In this short letter, we review the current status of theoretical calculation of semi-leptonic B decays, focusing on the method of light cone QCD sum rules. The correlation functions calculated using the light meson light cone distribution amplitudes (LCDAs) have been studied for some years, while sum rules using B meson light cone distribution amplitudes are studied later with effort only recently. The precision of both kinds of sum rules have been done up to the two loop QCD corrections and next-to-leading power. These improvements combining with the lattice QCD results at small recoil region, have significantly improve the theoretical precision of $|V_{ub}|$ and $|V_{cb}|$ determination.

The most important non-perturbative input in LCSR calculation is the meson light cone distribution amplitudes. Theoretical studies show that the main theoretical uncertainty in LCSR calculation is from the meson LCDAs [4]. There are some improvements in the study of hadron LCDAs, especially heavy meson LCDAS recently, including lattice QCD study.

2. Light cone sum rules and LCDAs

The semi-leptonic B decay rate depends on hadron transition form factors, which are functions of momentum square (q^2) of lepton pair. For large q^2 with small recoil of final state meson, lattice QCD can give very precise results for the form factors [3]. While for small q^2, we have to work on LCSR or various quark models. The LCSR is a QCD inspired method for the large recoil region of transition form factors. We can define the correlation functions by the light meson LCDA

$$\langle \pi(p)| T\{J_{weak}(0), J_\pi(x)\}|0\rangle, \tag{1}$$

or by the B meson LCDA as

$$\langle 0|T\{J_{weak}(0), J_B(x)\}|\bar{B}(p)\rangle. \tag{2}$$

In principle, we should get the same results from any of the above calculations. In the early years, people usually use the light meson LCDA as input, which was defined with better precision, especially for the ground state meson. Recently, the LCSR was usually done using the B meson LCDA, because it is universal for all B meson decays, which can reduce the overall theoretical uncertainty, especially for B decays to excited meson final states. We can calculate these correlation functions both at the quark level using operator product expansion and hadron level, which is proportional to the transition form factors. Applying quark hadron duality, and using Borel transformation to suppress the higher resonance contribution, we can get the form factors.

In quark level, the calculation can be done via α_s expansion by loop diagrams and via power expansion by effective theory, such as soft-collinear effective theory. The theoretical precision can be improved by high order calculation. Apparently, the largest theoretical uncertainty is from light cone distribution amplitudes (LCDAs) of hadrons. For light hadrons, such as pion, many studies give a reasonable precision; but worse precision for LCDA of excited meson states. While for heavy hadron, we have limited understanding of the heavy meson LCDAs. In fact, hadron LCDAs are also essential inputs in any of the factorization method of non-leptonic hadron decays [5, 6].

Unlike the light meson with large momentum as final state of B decays, the B meson is at rest, not on the light cone. We have to use the heavy quark effective theory to define its LCDA, and using models to describe its distribution function [7]. Recently, lattice QCD has made a great effort to calculate the LCDA of D and B meson [8, 9]. People make use of equal-time correlations, to simulate the so-called quasi-distribution amplitudes. Utilizing the large momentum effective theory, we get the QCD LCDA of heavy meson. By the multiplicative factorization form, we translate the QCD LCDA to the LCDA defined in heavy quark effective theory. This study will certainly reduce theoretical uncertainties of form factor study in LCSR.

3. $b \to c$ transition

$B \to D^{(*)}\ell\bar{\nu}_\ell$ decays are governed by $b \to c$ transition, which accounts for V_{cb} measurement. The branching ratios of these decays are proportional to $B \to D^{(*)}$ form factors, which are well defined in heavy quark effective theory. At the leading order of $1/m_b$ expansion, the form factors are reduced to the so called Isgur-Wise function with very small theoretical uncertainty. The lattice QCD can give $1/m_b$ power corrections with very high precision [3]. However, these conclusions are valid only for large momentum transfer q^2.

For small q^2, we have to rely on various quark model calculation or light cone sum rules calculation. The leading order LCSR calculation was done in 2009 [10]; while the next-to-leading order correction [11] and next-to-leading twist corrections [12] were performed later. A complete next-to-leading power correction was also performed [13]. Finally, an improved study on the $B \to D^{(*)}$ form factors including next-to-leading order correction together with next-to-leading power corrections were given [14]. As stated in the previous section, the LCSR calculation rely on the input of hadron LCDAs, which give the largest theoretical uncertainty. The CKM matrix element $|V_{cb}|$ extraction was updated by a global fit of q^2 dependence in the Boyd, Grinstein and Lebed parametrization [15], combining the recent lattice data, using experimental measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$ decays.

The average experimental results of

$$R(D^{(*)}) = \frac{Br(B \to D^{(*)}\tau\bar{\nu}_\tau)}{Br(B \to D^{(*)}\mu\bar{\nu}_\mu)}, \tag{3}$$

indicate about 3σ deviation from the standard model predictions [16]. Taking the deviation seriously, apparently tau lepton has a stronger coupling than that of muon or electron. Charged Higgs, seems a natural explanation but the simple models do not work, because of the conflict experimental results between $R(D)$ and $R(D^*)$. Extra new W gauge boson with non-universal couplings or leptoquark model with very specific flavour structure are proposed to explain these anomalies. Careful studies found that none of the single new physics operators can explain simultaneously the current experimental measurements of the ratios $R(D)$, $R(D^*)$ and $R(J/\Psi)$ at the confidence level of 1σ [17]. Even with 2σ Constraints, the scalar operators accounts for the charged Higgs contributions, are ruled out.

4. Heavy to light transition

The semi-leptonic decay $B \to \pi\ell\bar{\nu}_\ell$ is the simplest heavy to light process, which has been studied for some years. The experimental measurements for the inclusive $b \to u\ell\bar{\nu}_\ell$ decay has less precision than that of $b \to c\ell\bar{\nu}_\ell$ decay, due to the larger background. On the other hand, the final state pion has larger momentum than that of D meson, making the perturbative calculation of $B \to \pi\ell\bar{\nu}_\ell$ decay more reliable. The LCSR calculation using π meson distribution amplitudes, has been done to the twist-two level at next-to-leading order in α_s [18]. The next-to-next-to-leading order is partially computed in the large β_0 approximation [19].

Alternatively, the LCSR calculation for the $B \to \pi$ transition form factors has also been performed using the B meson distribution amplitudes [20]. Later, the next-to-leading logarithmic resummation improved sum rules for the $B \to \pi$ transition form factors were derived [21]. The QCD calculations of $B \to \pi, K$ form factors with higher-twist corrections were also given [22]. Very recently, the next-to-next-to-leading order QCD corrections to the $B \to \pi$ transition form factors from LCSR have been given [23]. With form factors at small q^2 by LCSR approach and that at large q^2 by lattice QCD, people usually use the Bourrely, Caprini and Lellouch parametrization [24] to get the full q^2 dependence. Finally the (differential) decay width of B meson semi-leptonic decays was obtained. Similar to the lepton universality ratio $R(D)$, $R(D^*)$, people define the ratio

$$R_\pi = \frac{Br(B \to \pi\tau\bar{\nu}_\tau)}{Br(B \to \pi\mu\bar{\nu}_\mu)}, \tag{4}$$

which is also sensitive to new physics.

The precision calculations of $B \to \rho, \omega, K^*$ form factors from soft-collinear effective theory sum rules on the light-cone are performed to the next-to leading order in α_s including sub-leading power corrections [4]. The next-to-leading order QCD corrections to B to scalar meson transition form factors from QCD LCSR with B meson light-cone distribution amplitudes was performed recently [25].

Unlike the tree level $B \to \pi(\rho)\mu\bar{\nu}_\mu$ decays, the $B \to K^{(*)}\ell^+\ell^-$ decay can occur only at loop level in the standard model, which are more sensitive to new physics contributions. The transition form factors governed these decays are penguin operators. A lot of studies have been performed over decades, including many studies of new physics contributions. There was ever an anomaly called $R(K^{(*)})$, which disappeared recently, due to the update of new experimental data. Theoretically, the next-to-leading-order weak annihilation correction to these rare decays have been done recently [26].

5. summary

In short, high precision theoretical study of semi-leptonic B decays are in progress. The transition form factors for these decays are calculated by lattice QCD at small recoil; and calculated by LCSR at large recoil. We review the current status of LCSR calculation of these form factors. Both of QCD loop corrections and power corrections for the transition form factors are performed, even with some of the annihilation type power corrections. The most important theoretical uncertainties are from the hadron LCDAs, which are poorly studied comparing with the perturbative corrections. The study of light hadron LCDAs are in progress and already in a considerable precision. While for the heavy hadron LCDAs, serious theoretical study is urgently needed.

The decay width and other experimental observables of semi-leptonic B decays are obtained combing the lattice QCD results and LCSR results. Some flavor anomalies have been discussed in the standard model and new phyiscs. We are still waiting for a clear New physics signal in the heavy flavor sector.

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