

A first study of strong isospin breaking effects in lattice QCD using truncated polynomials

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Computing derivatives of observables with respect to parameters of the theory is a powerful tool in lattice QCD, as it allows the study of physical effects not directly accessible in the original Monte Carlo simulation. Prominent examples of this include the impact of the up-down quark mass difference and electromagnetic corrections. In this work, we present a new approach based on automatic differentiation to evaluate such derivatives to arbitrarily high orders, where particular emphasis will be placed on strong isospin-breaking effects and on the propagation of derivatives through the conjugate gradient algorithm in the computation of correlation functions.

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1. Introduction

The computation of derivatives with respect to parameters of the action is a tool that has proven to be very useful in lattice QCD simulations. Among others, it allows to compute strong and electromagnetic isospin breaking effects—which contribute at the percent level to hadronic observables like hadron mass splittings and the HVP contribution to $g - 2$ —and also to correct for mistunings in the choice of the bare parameters in lattice simulations.

For the case of strong isospin breaking, the different methods to introduce its effects in lattice QCD simulations can be classified into two main groups:

- Exact methods, in which the exact isospin breaking contribution is included for both dynamic and valence quarks. This category includes, for example, performing explicit simulations with non-degenerate up and down quark masses or performing exact reweighting in an already existing isospin-symmetric ensemble.
- Approximate methods, in which isospin breaking is introduced as a Taylor expansion in powers of the up-down quark mass splitting Δm truncated to some power. This category includes the RM123 method [1], or performing measurements at different quark masses in order to extract derivatives by, for example, taking finite differences.

The focus of the present study regards the inclusion of strong isospin breaking effects using automatic differentiation through reweighting, which in essence will be a generalization and full automation of the RM123 method up to arbitrary orders.

This manuscript is organized as follows: in Sec. 2, we review the RM123 method to include strong isospin breaking effects, specializing for the case of the kaon correlation function; in Sec. 3, we explain how to generalize the RM123 method to arbitrary orders in Δm by introducing truncated polynomials along with reweighting; in Sec. 4, we explain the lattice setup of our simulation and show our results comparing both approaches; finally, we write our conclusions in Sec. 5.

2. Including isospin breaking effects with the RM123 method

Let us consider that we are interested in evaluating the expectation value

$$\langle O \rangle_{\Delta m} = \frac{\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} O e^{-S_0 - S_{\text{IB}}(\Delta m)}}{\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_0 - S_{\text{IB}}(\Delta m)}}, \quad (1)$$

in a theory with $N_f = 1 + 1$ non-degenerate quarks $\psi = [u, d]$ with an action that can be split into isospin-symmetric and isospin-broken parts,

$$S_0 = S_G + \sum_{f=u,d} \bar{\psi}_f D \psi_f + m_l \sum_x [\bar{u}u + \bar{d}d](x), \quad S_{\text{IB}}(\Delta m) = -\Delta m \sum_x [\bar{u}u - \bar{d}d](x), \quad (2)$$

where S_G is the gauge action, D is the massless Dirac operator, $m_l = (m_u + m_d)/2$ is the average of and $\Delta m = (m_d - m_u)/2$ the splitting between the masses m_u and m_d of the quarks u and d .

While simulating two degenerate quarks can be done efficiently in QCD with the standard Hybrid Monte Carlo algorithm [2], performing a simulation with non-degenerate quarks requires

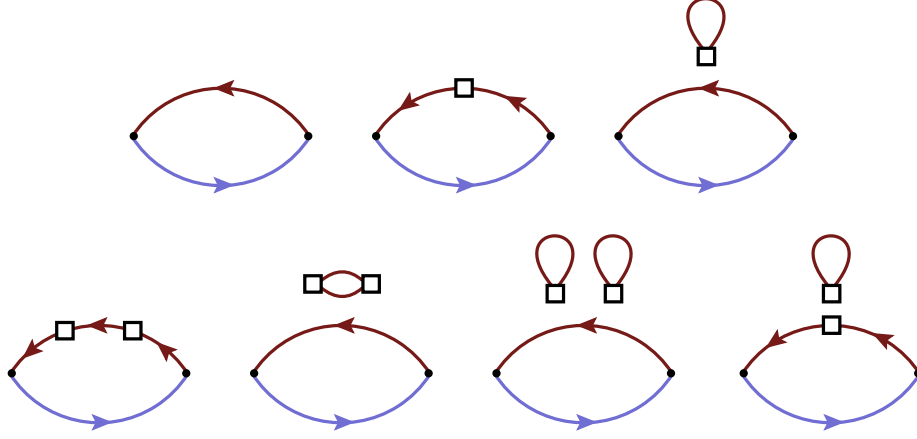


Figure 1: Diagrams resulting from evaluating the Wick contractions in Eq. (5) for the kaon correlation function in Eq. (7) to leading (top left), first order (top middle and top right), and second order (bottom row) in Δm . Filled black dots denote γ_5 insertions, empty squares denote insertions of S_{IB} , and red and purple lines denote up and strange quark propagator, respectively.

the use of more complex algorithms—such as the rational Hybrid Monte Carlo algorithm [3, 4]—which are computationally more expensive. The authors of Ref. [1] proposed as an alternative to approximate $\langle O \rangle_{\Delta m}$ by considering its Taylor expansion in Δm ,

$$\langle O \rangle_{\Delta m} = \langle O \rangle_0 + \left. \frac{\partial \langle O \rangle_{\Delta m}}{\partial \Delta m} \right|_{\Delta m=0} \Delta m + \mathcal{O}(\Delta m^2), \quad (3)$$

where $\langle O \rangle_0$ is an expectation value computed in the isospin-symmetric theory,

$$\langle O \rangle_0 = \frac{\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} O e^{-S_0}}{\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_0}}. \quad (4)$$

If one knows the derivative $\partial \langle O \rangle_{\Delta m} / \partial \Delta m \big|_{\Delta m=0}$ and Δm is small enough, one can approximate $\langle O \rangle_{\Delta m}$ by truncating at first order in Δm . Such derivative can be obtained by Taylor expanding the path integral in Eq. (1),

$$\begin{aligned} \langle O \rangle_{\Delta m} &= \frac{\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} O (1 + \Delta m S_{\text{IB}}) e^{-S_0}}{\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} (1 + \Delta m S_{\text{IB}}) e^{-S_0}} + \mathcal{O}(\Delta m^2) = \frac{\langle O \rangle_0 + \Delta m \langle O S_{\text{IB}} \rangle_0}{1 + \Delta m \langle S_{\text{IB}} \rangle_0} + \mathcal{O}(\Delta m^2) \\ &= \langle O \rangle_0 + \Delta m \langle O S_{\text{IB}} \rangle_0 + \mathcal{O}(\Delta m^2), \end{aligned} \quad (5)$$

where we have used that $\langle S_{\text{IB}} \rangle_0 = 0$ in the isospin-symmetric theory. By comparing with Eq. (3), we identify

$$\left. \frac{\partial \langle O \rangle_{\Delta m}}{\partial \Delta m} \right|_{\Delta m=0} = \langle O S_{\text{IB}} \rangle_0, \quad (6)$$

which can be evaluated in the cheaper, isospin-symmetric simulation.

In practice, this means that we can compute derivatives with respect to action parameters by computing additional diagrams. Specializing to the case of the charged kaon correlation function,

$$C_{K^+}(t) = \langle \bar{s}(t) \gamma_5 u(t) \bar{u}(0) \gamma_5 s(0) \rangle, \quad (7)$$

the leading order leads to the diagram in the top left of Fig. 1 and the first order to the two diagrams in top middle and right.¹ The number of diagrams to compute increases combinatorially with the truncation order of the Taylor expansion: if we were to truncate the Taylor expansion at second order, we would need to compute the four diagrams in the bottom of the figure, which need to be individually derived, implemented and tested. In the following, we will propose and test a framework, based on automatic differentiation, which fully automates the computation of all these diagrams up to arbitrary order.

3. Generalizing the RM123 method with truncated polynomials

3.1 Truncated polynomials

Truncated polynomials are a particular flavor of forward-mode automatic differentiation [5], and they rely on the fact that a differentiable function $f(x)$ can be expanded in a Taylor series around $x^{(0)}$. Particularly,

$$f(x^{(0)} + \Delta m) = f(x^{(0)}) + f'(x^{(0)})\Delta m + \frac{1}{2}f''(x^{(0)})\Delta m^2 + \dots \quad (8)$$

The construction of such Taylor expansion can be automated for any analytic function f by using the algebra of truncated polynomials: if $\tilde{x} = \tilde{x}^{(0)} + \tilde{x}^{(1)}\Delta m + \dots + \tilde{x}^{(K)}\Delta m^K$ is a truncated polynomial of order K and we code all elementary mathematical functions acting on these polynomials, and we evaluate $f(\tilde{x})$ with $\tilde{x} = \tilde{x}^{(0)} + \Delta m$, the output of the function will be a truncated polynomial containing its first K derivatives

$$f^{(k)}(\tilde{x}^{(0)}) = \frac{1}{k} \frac{\partial^k f(x)}{\partial x^k} \Big|_{x=\tilde{x}^{(0)}}. \quad (9)$$

This machinery can be used, in principle, for any arbitrarily complex function f , such as a computer program. Particularly, it has been successfully applied in lattice QCD for automatic error propagation in the analysis of Monte Carlo data [6], as well as in lattice field theory simulations to propagate derivatives with respect to action parameters throughout the HMC algorithm or via reweighting techniques [7], to tackle the sign problem in the quantum rotor [8], and to tackle the signal to noise problem in a ϕ^4 theory [9].

In the present study we want to test if automatic differentiation can successfully propagate derivatives when solving the Dirac equation, which is key for the computation of matrix elements and hadron masses from correlation functions such as the one in Eq. (7). As in the previously mentioned applications, we will use the algebra of truncated polynomials as implemented in `FormalSeries.jl` [10] in the Julia programming language, and focus on the computation of observables via reweighting.

3.2 Reweighting with truncated polynomials

As mentioned in the introduction, a possible way to introduce isospin breaking effects in an observable is by reweighting an ensemble from $\Delta m = 0$ to a particular $\Delta m \neq 0$ [11, 12]. To do so,

¹The top right diagram cancels out when summing the contributions coming from the up and down quark [1].

we write

$$\begin{aligned}\langle O \rangle_{\Delta m} &= \frac{\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} O e^{-S_0 - S_{\text{IB}}(\Delta m)}}{\int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_0 - S_{\text{IB}}(\Delta m)}} = \frac{\int \mathcal{D}U O(\Delta m) W_{\text{IB}}(\Delta m) e^{-S_G}}{\int \mathcal{D}U W_{\text{IB}}(\Delta m) e^{-S_G}} \\ &= \frac{\langle O(\Delta m) W_{\text{IB}}(\Delta m) \rangle_0}{\langle W_{\text{IB}}(\Delta m) \rangle_0},\end{aligned}\quad (10)$$

where in the second equality we have performed the integration over fermionic fields and we have defined the isospin-breaking reweighting factor

$$W_{\text{IB}}(\Delta m) \equiv \frac{\det D(m_l - \Delta m) \det D(m_l + \Delta m)}{\det D^2(m_l)},\quad (11)$$

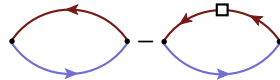
and expectation values on the right-hand side are evaluated in the isospin-symmetric theory. If we replace Δm by the truncated polynomial

$$\Delta \tilde{m} = \sum_{k=0}^K \Delta \tilde{m}^{(k)} \Delta m, \quad \Delta \tilde{m}^{(k)} = \begin{cases} 1 & \text{for } k = 1 \\ 0 & \text{for } k \neq 1 \end{cases},\quad (12)$$

evaluating Eq. (10) will return the full analytical dependence of the polynomial expansion of the observable O up to order K centered around $\Delta m = 0$,

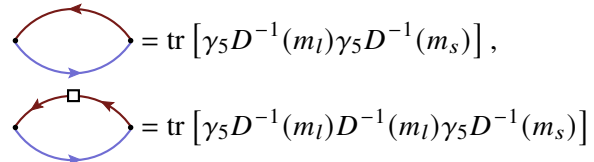
$$\langle O \rangle_{\Delta m}^{(k)} = \frac{1}{k!} \frac{\partial^k}{\partial (\Delta m)^k} \langle O \rangle_{\Delta m} \Big|_{\Delta m=0}.\quad (13)$$

The evaluation of Eq. (10) with truncated polynomials requires, on the one hand, the computation of $O(\Delta \tilde{m})$, which accounts for the valence contribution of isospin-breaking effects. In our study we will focus on the correlation function of the kaon, which, after integrating out fermionic degrees of freedom, reads

$$O(\Delta m) = \text{tr} [\gamma_5 D^{-1}(m_l - \Delta m) \gamma_5 D^{-1}(m_s)] = - \text{diag}_1 - \text{diag}_2 \Delta m + O(\Delta m^2),\quad (14)$$


where the dependence of the Dirac operator on the quark masses is displayed for clarity and we have explicitly shown the diagrammatic expansion of the observable up to first order in Δm . While in the RM123 method one needs to explicitly compute both diagrams by implementing

$$\text{diag}_1 = \text{tr} [\gamma_5 D^{-1}(m_l) \gamma_5 D^{-1}(m_s)],\quad (15)$$

$$\text{diag}_2 = \text{tr} [\gamma_5 D^{-1}(m_l) D^{-1}(m_l) \gamma_5 D^{-1}(m_s)],\quad (16)$$


the use of truncated polynomials allows to obtain the full polynomial expansion of the observable by the evaluation of the left-hand side of Eq. (14) with already existing code.

On the other hand, the computation of the reweighting factor W_{IB} accounts for the sea-quark contribution of isospin-breaking effects, and can also be evaluated using the truncated polynomial

in Eq. (12). Since $\Delta\tilde{m}^{(0)} = 0$, the ratio in Eq. (11) is 1 at leading order and can be easily estimated using N complex normal stochastic sources η_i :

$$W_{\text{IB}} \equiv \det A = \int d\eta e^{-\eta^\dagger A^{-1} \eta} = \int d\eta e^{-\eta^\dagger \eta} e^{-\eta^\dagger (A^{-1}-1)\eta} \approx \frac{1}{N} \sum_{i=1}^N e^{-\eta_i^\dagger (A^{-1}-1)\eta_i}, \quad (17)$$

where $A \equiv D_u D_d D_l^{-2}$. Note that, once estimated, W_{IB} can be reused for any other observable $O(\Delta m)$.

In the following, we will focus on the computation of the valence contribution of the kaon correlator in Eq. (14) with truncated polynomials, where we will solve the Dirac equation by means of the conjugate gradient algorithm.

3.3 Conjugate gradient with truncated polynomials

The conjugate gradient algorithm is an iterative algorithm that solves the Dirac equation

$$D\psi = \eta. \quad (18)$$

Although truncated polynomials can be used with any computer program as long as all mathematical operations acting on them are differentiable, it is not clear that the propagated derivatives will converge as rapidly, nor if this is guaranteed, for an iterative algorithm such as the conjugate gradient. The main objective of this study is to investigate whether truncated polynomials converge to the right derivatives using the conjugate gradient algorithm by comparing results with the RM123 method.

The only step which is non-differentiable in the conjugate gradient algorithm and should be adapted to the use of automatic differentiation is the choice of the stopping criterium. In the standard conjugate gradient, the usual choice is to enforce that the residue of the solution is smaller than a particular tolerance `tol`, i.e.

$$\|(DD^{-1} - I)\eta\|^2 < \text{tol} \|\eta\|^2. \quad (19)$$

However, when using truncated polynomials what we will have is a Taylor expansion in Δm of our fermion fields,

$$\eta \rightarrow \tilde{\eta} = \tilde{\eta}^{(0)} + \tilde{\eta}^{(1)} \Delta m + \tilde{\eta}^{(2)} \Delta m^2 + O(\Delta m^3). \quad (20)$$

In particular, the residue of the solution will also become a truncated polynomial: expanding $DD^{-1} - I$ in Δm , we have

$$DD^{-1} - I - \Delta m [DD^{-2} - D^{-1}] + \Delta m^2 [DD^{-3} - D^{-2}] + O(m^3). \quad (21)$$

By analogy with the condition for the leading order in Eq. (19), we decided to enforce that the residue is smaller than a certain tolerance `tol` order by order,²

$$\|DD^{-n-1}\eta - D^{-n}\eta\| < \text{tol} \|\eta\|. \quad (22)$$

²Note that this stopping criterium is not the same as imposing $\|[(DD^{-1} - I)\eta]\|^2 < \text{tol} \|\eta\|^2$ for every order. Such an expansion up to order Δm would be

$$([DD^{-1} - I]\eta, [DD^{-1} - I]\eta) - \Delta m \{([DD^{-2} - D^{-1}]\eta, [DD^{-1} - I]\eta) + ([DD^{-1} - I]\eta, [DD^{-2} - D^{-1}]\eta)\} + O(\Delta m^2)$$

which mixes contributions from different orders of the expansion of $DD^{-1} - I$.

ID	β	BC	V	a	M_π	M_K	$M_\pi L$
A654	3.34	periodic	48×24^3	0.097 fm	338 MeV	462 MeV	4.0

Table 1: Parameters of the simulation used in this study: the bare coupling $\beta = 6/g_0^2$, the boundary condition, the volume in lattice units, the lattice spacing a in physical units, and the approximate pion and kaon masses.

4. Setup and results

In the following, we will investigate whether the condition in Eq. (22) leads to a convergence of the first order in Δm when using truncated polynomials with the conjugate gradient algorithm. For this proof-of-concept study we used a single ensemble, generated using `openQCD` [13] as part of the Coordinated Lattice Simulations (CLS) initiative [14]. It has $N_f = 2 + 1$ flavors of $O(a)$ -improved Wilson fermions and a tree-level improved Lüscher–Weisz gauge action, and other parameters of its simulation are displayed in Tab. 1.

We focused on the computation of the derivative of the kaon mass with respect to the up-down mass splitting, $\partial M_{K^+}/\partial \Delta m$, for which we evaluated Eq. (14) using truncated polynomials. For this purpose, we used a code designed to perform lattice simulations on GPUs, `LatticeGPU.jl` [15, 16], written in Julia, which has I/O compatibility with `openQCD`. Additionally, we used the measurement code `GPUobs.jl` [17], which allows one to obtain observables as truncated polynomials in Δm , and we set the solver tolerance `tol` in Eq. (22) to 10^{-28} .

By evaluating Eq. (14) with truncated polynomials of order 1, one obtains the power expansion

$$\tilde{C}_{K^+}(t) = C_{K^+}^{(0)} + C_{K^+}^{(1)}\Delta m + \mathcal{O}(\Delta m^2). \quad (23)$$

To extract the derivative of the kaon mass, we start by writing the large-time behavior of the spectral decomposition of the kaon two-point function,

$$C_{K^+}(t) \propto A \left[e^{-M_{K^+}(t-T/2)} + e^{M_{K^+}(t-T/2)} \right]. \quad (24)$$

By expanding the matrix element and the kaon mass in powers of Δm ,

$$\tilde{A} = A^{(0)} + A^{(1)}\Delta m + \mathcal{O}(\Delta m^2), \quad \tilde{M}_{K^+} = M_{K^+}^{(0)} + M_{K^+}^{(1)}\Delta m + \mathcal{O}(\Delta m^2) \quad (25)$$

one can obtain $M_{K^+}^{(1)}$ by fitting to the relation

$$\frac{C_{K^+}^{(1)}(t)}{C_{K^+}^{(0)}(t)} = \frac{A^{(1)}}{A^{(0)}} + M_{K^+}^{(1)}(t - T/2) \tanh[M_{K^+}^{(0)}(t - T/2)]. \quad (26)$$

In Fig. 2 (left), we show this ratio as a function of time, where the data was obtained from a set of 100 configurations of the ensemble A654, and fitting to the previous functional form we find

$$M_{K^+}^{(1)} = -3.86(52). \quad (27)$$

As a cross-check of this method, in Fig. 2 (right) we plot the relative difference between the derivative $C_{K^+}^{(1)}(t)$ obtained with automatic differentiation and the one obtained with the RM123,

$$\frac{\Delta C^{(1)}(t)}{C_{\text{RM123}}^{(1)}(t)} \equiv \frac{C_{K^+, \text{RM123}}^{(1)}(t) - C_{K^+, \text{AD}}^{(1)}(t)}{C_{K^+, \text{RM123}}^{(1)}(t)} \quad (28)$$

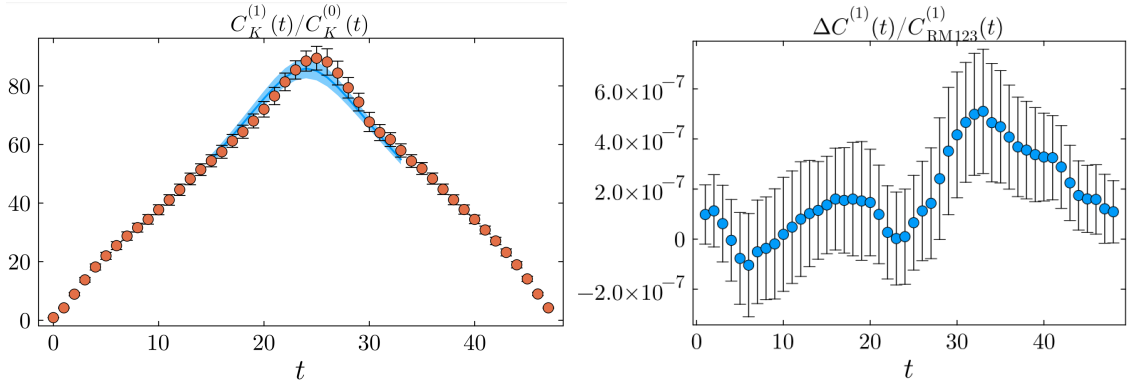


Figure 2: (Left) Ratio of first to leading order of correlation function of the kaon as a function of time. The blue band denotes the region fitted to the functional form in Eq. (26). (Right) Relative difference between the first order of the correlation function of the kaon obtained with truncated polynomials and RM123, and defined in Eq. (28).

where $C_{K^{+1},\text{RM123}}^{(1)}$ is obtained by evaluating Eq. (16). The fact that this relative difference is of the order of 10^{-7} is an indication that derivatives with automatic differentiation can be correctly propagated through the conjugate gradient algorithm, and that the RM123 method up to arbitrary order can be fully automated with truncated polynomials.

5. Conclusions and outlook

We have developed a new method to compute derivatives with respect to any action parameter up to arbitrary order using truncated polynomials. Using this method, we have studied the valence contribution of strong isospin breaking effects of the kaon correlation function at first order in Δm . As a first proof of concept, we have found that the first derivative of the kaon correlation function computed with truncated polynomials converges and is compatible with the one obtained with the RM123. A more thorough assessment of the solution's precision and a comparison of the computational cost with respect to the RM123 method is currently a work in progress.

In the present study, we have restricted ourselves to the valence contribution at first order in Δm . A natural next step is to extend the analysis to higher orders in the mass expansion and to incorporate sea-quark contributions by computing the reweighting factor in Eq. (11). Also, although we have focused on expansions in Δm to describe strong isospin-breaking effects, the framework is fully general and can be applied to any action parameter, and could be used, among others, for the correction of mistunings in the quark masses and for the inclusion of electromagnetic isospin-breaking effects.

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References

- [1] G.M. de Divitiis et al., *Isospin breaking effects due to the up-down mass difference in Lattice QCD*, *JHEP* **04** (2012) 124 [1110.6294].
- [2] S. Duane, A.D. Kennedy, B.J. Pendleton and D. Roweth, *Hybrid Monte Carlo*, *Phys. Lett. B* **195** (1987) 216.
- [3] A.D. Kennedy, I. Horvath and S. Sint, *A New exact method for dynamical fermion computations with nonlocal actions*, *Nucl. Phys. B Proc. Suppl.* **73** (1999) 834 [hep-lat/9809092].
- [4] M.A. Clark and A.D. Kennedy, *The RHMC algorithm for two flavors of dynamical staggered fermions*, *Nucl. Phys. B Proc. Suppl.* **129** (2004) 850 [hep-lat/0309084].
- [5] A. Haro, *Automatic differentiation tools in computational dynamical systems*, .
- [6] A. Ramos, *Automatic differentiation for error analysis of Monte Carlo data*, *Comput. Phys. Commun.* **238** (2019) 19 [1809.01289].
- [7] G. Catumba, A. Ramos and B. Zaldivar, *Stochastic automatic differentiation for Monte Carlo processes*, *Comput. Phys. Commun.* **307** (2025) 109396 [2307.15406].
- [8] D. Albandea, G. Catumba and A. Ramos, *Strong CP problem in the quantum rotor*, *Phys. Rev. D* **110** (2024) 094512 [2402.17518].
- [9] G. Catumba and A. Ramos, *Stochastic automatic differentiation and the signal to noise problem*, *Eur. Phys. J. C* **85** (2025) 1037 [2502.15570].
- [10] A. Ramos, *Formalseries.jl*, May, 2023. 10.5281/zenodo.7970278.
- [11] J. Finkenrath, F. Knechtli and B. Leder, *One flavor mass reweighting in lattice QCD*, *Nucl. Phys. B* **877** (2013) 441 [1306.3962].
- [12] J. Finkenrath, F. Knechtli and B. Leder, *Isospin Effects by Mass Reweighting*, *PoS LATTICE2014* (2015) 297 [1501.06441].
- [13] M. Lüscher, “Charm and strange quark in openQCD simulations.” <http://luscher.web.cern.ch/luscher/openQCD/>, 2019.
- [14] M. Bruno, T. Korzec and S. Schaefer, *Setting the scale for the CLS 2 + 1 flavor ensembles*, *Phys. Rev. D* **95** (2017) 074504 [1608.08900].

- [15] G. Catumba, F. Pérez Panadero, C. Pena and A. Ramos, “latticegpu.jl.”
<https://igit.ific.uv.es/alramos/latticegpu.jl>, 2024.
- [16] G. Catumba, F. Pérez Panadero, C. Pena and A. Ramos, *A Julia Code for Lattice QCD on GPUs*, *PoS LATTICE2024* (2025) 281 [2503.16994].
- [17] G. Catumba, F. Pérez Panadero, C. Pena and A. Ramos, “gpuobs.jl.”
<https://igit.ific.uv.es/alramos/gpuobs.jl>, 2024.
- [18] D. Djukanovic, *Quark Contraction Tool — QCT*, *Comput. Phys. Commun.* **247** (2020) 106950 [1603.01576].