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Mixing of light meson and charmonium flavor singlets in $N_f = 3 + 1$ QCD

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We investigate the mixing between S-wave flavor-singlet light meson and charmonium operators in two ensembles at different pion masses ($m_\pi \approx 420, 800$ MeV). By solving a GEVP we find both types of operators have non-zero overlaps with all states we look at. We also compare the resulting spectrum with the one coming from separate GEVPs including either only light meson or only charmonium operators and quantify the effects of the mixing on the hyperfine splitting. The largest effect we observe is a decrease of the η_c mass by 39(24) MeV. Finally, we show preliminary results of the mixing between both types of operators with glueball ones as a first step to further extend our operator basis.

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1. Introduction

Experimental and theoretical interest in exotic states, particularly the so-called XYZ states [1], has been renewed in part thanks to the recent identification of the $X(2370)$ as a pseudo-scalar glueball candidate [2]. These states are also approached in lattice calculations, where they can be systematically studied from first principles. In these calculations, flavor-singlets such as glueballs, charmonium and some light mesons are in the same symmetry channel once all other lattice quantum numbers have been fixed, e.g. total lattice angular momentum, parity and charge conjugation. Because of this, the correlation function built from a flavor singlet operator, e.g. charmonium, receives contributions in its spectral decomposition from energy eigenstates with the same quantum numbers including those above and below the charmonium energy region. This can be written explicitely as

$$\begin{aligned} C_{\bar{c}c}(t) &= \langle \bar{c}(t)c(t) \cdot \bar{c}(0)c(0) \rangle \\ &= |\langle \eta' | \, \bar{c}c \, | \Omega \rangle|^2 e^{-m_{\eta'} t} + |\langle \eta'(2S) | \, \bar{c}c \, | \Omega \rangle|^2 e^{-m_{\eta'(2S)} t} + \ldots \\ &+ |\langle \eta_c | \, \bar{c}c \, | \Omega \rangle|^2 e^{-m_{\eta_c} t} + |\langle \eta_c(2S) | \, \bar{c}c \, | \Omega \rangle|^2 e^{-m_{\eta_c(2S)} t} + \ldots, \end{aligned} \tag{1}$$

where we consider pseudoscalar quantum numbers, i.e. $R^{PC} = A_1^{-+}$, and only write down contributions from single-particle states, although in principle any multi-particle state with matching quantum numbers has a non-zero contribution even if significantly suppressed. While the OZI rule predicts $|\langle \eta' | \, \bar{c}c \, | \Omega \rangle|^2 << |\langle \eta_c | \, \bar{c}c \, | \Omega \rangle|^2$ [3], no symmetry enforces $\langle \eta' | \, \bar{c}c \, | \Omega \rangle|^2 = 0$ and the correlation function is asymptotically dominated by the contribution from the η', not the η_c, at large enough temporal separation. This will be the case for any operator with these quantum numbers because charmonium is not the ground state in this channel. Since in this work we are interested in calculating the charmonium spectrum in a setup where it can mix with lighter flavor singlets, we will build a basis of operators which accounts for the different types of single-particle states contained in the ladder of excitations. By using these operators in a GEVP formulation, we can resolve the different states of interest while at the same time gaining insight on their composition in terms of our operator basis to match them with the corresponding light meson and charmonium energy eigenstates [4].

2. Methods

We work in an $N_f = 3+1$ ensemble with $m_\pi \approx 420$ MeV, lattice size 96×32^3, $\beta = 3.24$ and $a \approx 0.05198(36)$ fm, three clover-improved Wilson degenerate light quarks at the $SU(3)$ flavor symmetric point with a close-to-physical charm quark, Lüscher-Weisz gauge action and open boundary conditions in time [4–7]. To build charmonium and light meson operators which best overlap onto the different energy eigenstates of interest we use the framework of optimal distillation profiles [8, 9]. We use $N_v = 200$ distillation vectors for the charm quark and $N_v = 100$ for the light quarks measured sufficiently far away from the temporal boundaries, all calculated using 3D APE-smeared gauge links [10] via a Thick-Restart Lanczos algorithm also used in [8], and the corresponding perambulators were calculated using the solvers available in the open-source package openQCD version 1.6 [11]. To account for the mixing, we build a correlation matrix between light meson $\bar{l}l$ and charmonium $\bar{c}c$ operators as

$$C(t) = \begin{pmatrix} \langle O_{\bar{l}l}(t)\bar{O}_{\bar{l}l}(0)\rangle & \langle O_{\bar{l}l}(t)\bar{O}_{\bar{c}c}(0)\rangle \\ \langle O_{\bar{c}c}(t)\bar{O}_{\bar{l}l}(0)\rangle & \langle O_{\bar{c}c}(t)\bar{O}_{\bar{c}c}(0)\rangle \end{pmatrix}, \tag{2}$$

where the diagonal entries involve both quark-connected and quark-disconnected contributions. While the latter are fundamental to distinguish between the pion and the η' in the light meson sector, they are often neglected in charmonium. In this work we take them into account, since the *implicit* mixing between light mesons and charmonium happens through these contributions [12]. The off-diagonal entries involve only quark-disconnected contributions and represent *explicit* mixing as they explicitly combine both types of operators. Each entry in Eq. 2 is a block containing multiple light meson and charmonium operators as to account for different radial excitations. It is worth noting that some multi-hadron states ($\pi\pi\pi$, $f_0\pi$, etc...) are expected between the η' and the η_c and therefore multi-hadron operators are required to account for them, which we do not include in this study. While the different quark-disconnected contributions are necessary to correctly sample the flavor singlet channel, they suffer from a severe signal-to-noise problem and we attempt to remedy this by using considerably large statistics with 9000 gauge configurations. We also do partial pruning on the matrix, i.e. out of the 7 light meson and 7 charmonium operators we choose 5 from each, corresponding to the singular vectors with largest singular values at a fixed reference time [4]. This way the GEVP is better conditioned while the light meson and charmonium operators are kept separate.

3. Results

We used the correlation matrix as defined in Eqn. 2 to extract the low-lying spectrum in two ways. First we solve two different GEVPs; one with only charmonium operators and one with only light meson operators, both including their corresponding quark-disconnected contributions. This is equivalent to eliminating the explicit mixing. Second, we solve a GEVP including the full correlation matrix. In this second case we account for explicit mixing, which we found to be statistically non-zero and the mixing correlations even change sign as a function of the temporal separation. In Fig. 1 we show the resulting spectrum, from where it is clear the non-zero mixing leaves the spectrum unchanged. This is opposite to what we previously observed for the A_1^{++} channel, where already including implicit mixing shifted the charmonium-only masses down to the light meson energy region [13]. The stability of the spectrum is not our only measure of the mixing effects; from the GEVP vectors we can extract the overlaps between the states created by our different operators and the energy eigenstates [14]. Fig. 2 shows the resulting overlaps (in absolute value) for each operator onto each of the relevant energy eigenstates. Due to unknown operator-dependent renormalization factors, we can only compare overlaps of a same operator onto different energy eigenstates. We observe non-zero overlaps between states created by charmonium (light meson) operators and the light meson (charmonium) energy eigenstates, which further confirms the non-zero contributions of the different types of states to the spectral decomposition of the correlation functions of the different types of operators. The charmonium operators have a strong preference for the states we label as η_c and $\eta_c(2S)$, with relatively small overlaps with the ones we label η' and $\eta'(2S)$. The light meson operators have a stronger preference to the η' and $\eta'(2S)$ and the overlaps with charmonium states are relatively larger than the ones of charmonium operators with light meson states. This is particularly true for $\bar{l}l_2$, $\bar{l}l_3$ and $\bar{l}l_4$. Since these come from pruning vectors which mostly align with higher excitations in the charmonium energy region, these larger overlaps are not unexpected.

We also look at the effects of the mixing on a more sensitive benchmark quantity; the hyperfine splitting. This quantity is usually measured in the lattice using only quark-connected charmonium correlations and it is important to quantify the effects of including the quark-disconnected contributions. For the J/ψ we found a negligible effect from the quark-disconnected contributions [4], which is expected due to a 3-gluon OZI suppression [3], yet for the η_c we found a non-negligible effect in the effective mass. We quantify this effect in two ways. First, a measurement of effective masses with and without the quark-disconnected contribution to compare the plateau. While this is the most direct approach, it relies on reaching a reliable plateau before the statistical errors grow too much. Second, we define the ratio

$$R(t) = \frac{C_{\text{Full}}(t)}{C_{\text{Conn}}(t)} \tag{3}$$

$$\overset{t\to\infty}{\approx} Ae^{-(m_{\text{Full}}-m_{\text{Conn.}})t}, \tag{4}$$

where $C_{\text{Full}}(t)$ and $C_{\text{Conn}}(t)$ are the correlations for the η_c with and without quark-disconnected contributions. When including the disconnected contributions, we also distinguish between the cases with and without explicit mixing. In the second line we have assumed $C_{\text{Conn}}(t)$ has a spectral decomposition dominated by a leading exponential term, which is not the case without quark-disconnected contributions in our dynamical $N_f = 3 + 1$ setup however this proved a good approximation when setting the scale for this ensemble [4]. With this caveat in mind, the advantage of this approach is we can extract the mass shift as a correlated difference benefiting from all points. Previous lattice studies determined the sign of the shift from the slope of $R(t) - 1$ [15–17]. In Fig. 3 we show the results for the mass shift from these different determinations. The effective masses extracted from Eqn. 4 are consistent with the mass plateau difference at large enough temporal separation, from where we report a shift of −37(15) MeV with only implicit mixing and −39(24) MeV with both implicit and explicit mixing. While this is consistent with previous similar determinations [15–17] and has the same sign as predicted from NRQCD perturbation theory [18, 19], it contrasts with recent lattice determination which put this value around 3 – 7 MeV [20, 21]. Since the first study is an indirect measurement, the second one works at different quark masses and number of flavors and we are within 2σ of a sign change, we need higher statistical precision before making a strong statement and simply emphasize how the improved distillation machinery allows to take into account the different types of mixing relevant for this work. We also directly measured the mixing between our meson operators and spatial Wilson loops, which can help account for any glueball-like states in the spectrum. While we observed statistically non-zero correlations, the signal was lost to noise earlier than any of the plateaus on which we based our analysis. A reliable inclusion of these and other purely gluonic operators will require dedicated methods to tackle their inherent signal-to-noise problem, such as multi-level approaches which have been systematically studied recently in pure gauge as well as quenched setups [22, 23].

A straightforward improvement to our calculation is to extend our operator basis to achieve a faster convergence to a reliable mass plateau. We present here for the first time a combination of optimal distillation profile together with a basis of multiple derivative-based meson operators, applied first to charmonium with only quark-connected correlations where the signal is clearest and we can test this improvement, using A_1^{++}. We use the operators $\bar{c}\mathbb{I}c$, $\bar{c}\gamma_i\nabla_i c$ and $\bar{c}\gamma_4\gamma_5\gamma_i\mathbb{B}_i c$, the last two taken from [14]. In Fig. 4 we show the ground state effective masses obtained in different ways. First, solving a GEVP using standard distillation and only the three choices of operators listed above, labeled as "3 Ops.". Second, solving a GEVP using the three operators above with 7 distillation profiles each, labeled as "All". Third, solving separate GEVPS for each operator with 7 profiles, labeled as "$\mathbb{I}$ + Profs", "$\gamma_i\nabla_i$ + Profs" and "$\gamma_4\gamma_5\gamma_i\mathbb{B}_i$ + Profs". The first feature of interest is the significant improvement going from using standard distillation with three operators to distillation profiles combined with the same three operators. At our choice of $N_v = 200$, the plateau region could become around twice as long. The second feature is the already large improvement which comes from going from standard distillation with three operators to distillation profiles with one operator, particularly $\mathbb{I}$. In this case the effective masses are already very close to the ones of the full operator basis and do not require the calculation of derivative elementals. This shows the already known advantage of distillation profiles to very cheaply extend the operator basis compared to using a large number of derivative-based operators. To explore this advantage, we show in Fig. 5 the first few excitations in this charmonium channel determined in two different ways: the 3×3 GEVP involving the three operators with standard distillation (labeled as HadSpec i, $i = 0, 1, 2$) and the 21×21 using those same three operators but with seven distillation profiles each one (labeled as HadSpec + Prof i, $i = 0, 1, 2, ..., 6$). While both can resolve the ground state, the following excitations are not clearly resolved by the 3×3 GEVP, while the other GEVP, where the only difference is the inclusion of distillation profiles, can clearly resolve a ladder of excitations. This is a very promising result in our setup, as it shows we can get access to a variety of excitations without needing too many or too complicated derivative elementals and can extend the basis with profiles.

4. Conclusions and Outlook

In this work we presented a direct measurement of the mixing between charmonium and light meson operators by accounting for quark-disconnected contributions to the correlation matrix where this can happen. Statistically non-zero signal for mixing correlations between both types of operators confirm they do not fully decouple. This mixing is relatively small and the extracted spectrum remains consistent with or without these effects. This is contrary to the significant mixing effects we saw in our study of the scalar (A_1^{++}) channel in this same ensemble. These results show how this mixing should ideally be explicitly measured in different channels instead of simply assumed small, as the magnitude of the effects is channel-dependent. The largest effect we observed was a negative shift of the η_c mass by 39(24) MeV, which is consistent with some previous studies yet we need higher precision to make a stronger statement. We also calculated the mixing with glueball-like operators based on spatial Wilson loops, however the signal is lost before the plateau regions on which we base our analysis. The signal-to-noise problem of glueball-like operators makes further improvements necessary, e.g. multi-level sampling [22, 23]. As a first step towards an improvement of our measurements, we presented for the first time results of extending our basis of operators by using multiple derivative-based ones together with optimal distillation profiles. We tested this for the A_1^{++} channel using only quark-connected charmonium correlations. This

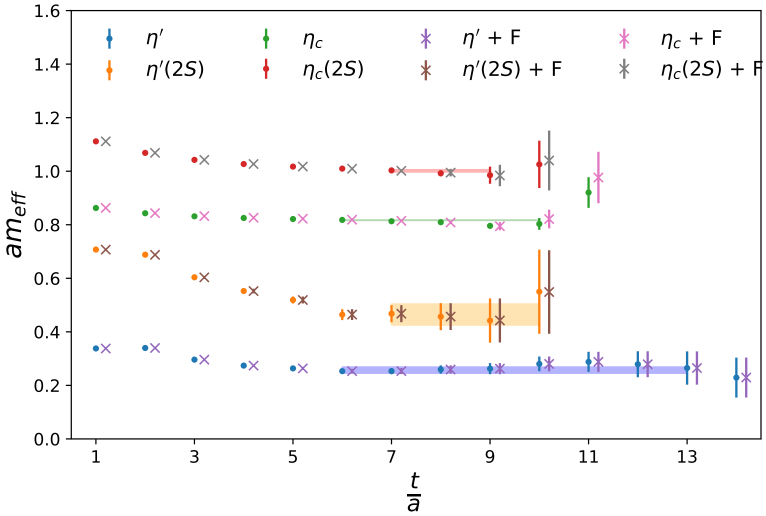

Figure 1: Lowest two states in the light meson and charmonium energy regions which we identify with the η', $\eta'(2S)$, η_c and $\eta_c(2S)$ coming from two approaches. **Approach 1:** Calculate η', $\eta'(2S)$ (η_c, $\eta_c(2S)$) from a GEVP involving only light meson (charmonium) operators only. **Approach 2:** Calculate these states from a single GEVP involving both types of operators. These are the states including "$+F$" in their labels.

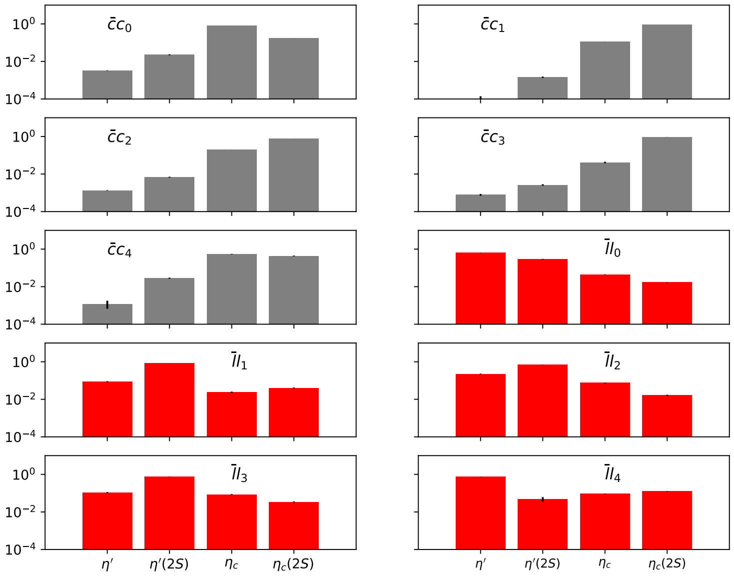

Figure 2: Overlaps of the states created by the charmonium $\bar{c}c_i$ and light meson $\bar{l}l_i$ operators onto the energy eigenstates of interest.

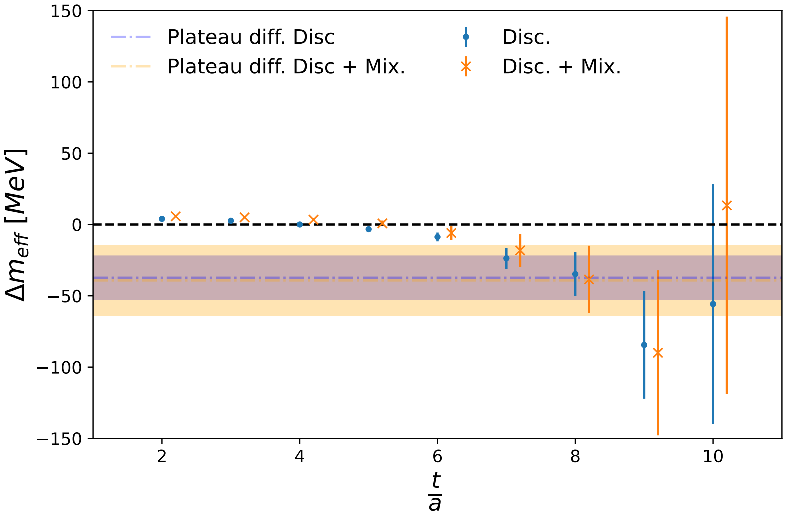

Figure 3: Mass shift of η_c due to the inclusion of quark-disconnected contributions. The dots and ×'s are the effective masses extracted from Eqn. 4 with and without the additional effect of including explicit mixing ("+Mix"). The bands are the mass shift calculated from the difference between the respective mass plateaus.

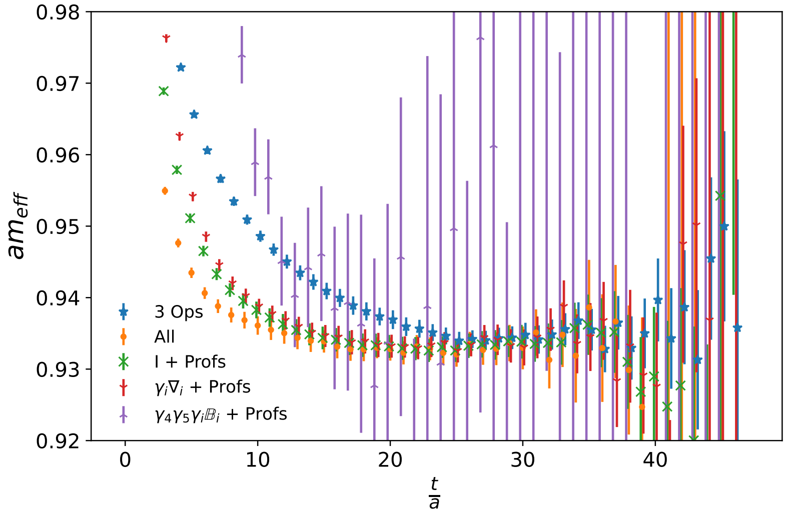

Figure 4: Ground state effective masses for the A_1^{++} charmonium with only quark-connected correlations, extracted from GEVPs involving: $\bar{c}\mathbb{I}c$, $\bar{c}\gamma_i\nabla_i c$ and $\bar{c}\gamma_4\gamma_5\gamma_i\mathbb{B}_i c$ with standard distillation (blue stars), these three operators all together with seven profiles each one (orange dots) and each operator separately with seven profiles.

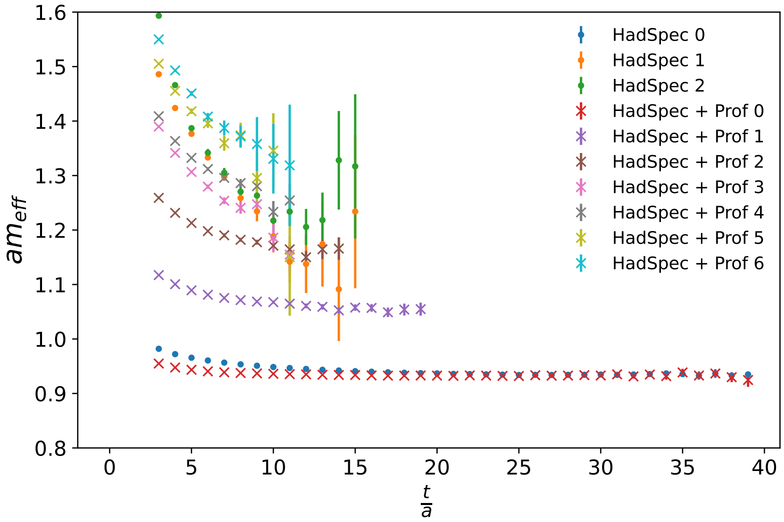

Figure 5: Effective masses for the A_1^{++} charmonium with only quark-connected correlations coming from two separate GEVPs: one with the three operators using standard distillation and another one using the same three operators with 7 distillation profiles each one.

extension made higher excitations available as well as revealed high-lying intermediate ones which our original basis did not resolve. While this did not affect our results on the low-lying spectrum, it displays the advantages of using derivative-based operators which can couple to states dominated by higher continuum J [14]. By comparing the spectrum obtained from these derivative-based operators with standard distillation and the spectrum coming from only one operator with optimal profiles, we saw the latter yields a faster convergence of the ground state to a plateau as well as a clear ladder of radial excitations which the former cannot resolve. This further shows the usefulness and flexibility of the optimal distillation profiles approach and we plan on applying this to further improve our upcoming calculation.

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