

Three-body study of the $T_{cc}(3875)^+$ from lattice QCD

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We discuss an ongoing first lattice study of the doubly-charmed tetraquark $T_{cc}^+(3875)$ via a three-body approach. We investigate the $DD\pi$ system in the $I = 0$, $C = 2$ sector, where the T_{cc}^+ appears as a pole in the $J^P = 1^+ DD\pi$ elastic scattering amplitude. The approach automatically incorporates two-body DD^* and three-body $DD\pi$ effects and treats left-hand cuts due to single π exchanges. Two CLS ensembles, X252 and X253, with pion mass $M_\pi \approx 280$ MeV, are used, and an operator set comprised of two- and three-hadron and tetraquark operators is employed to extract finite-volume energies. Additional inputs are required for the three-body finite-volume analysis, in the form of amplitudes for the $I = 1 DD$ and $I = 1/2 D\pi$ two-body subsystems. We present preliminary results for these subchannels and perform exploratory three-body spectra determinations for simple choices of the three-particle K-matrix $\mathcal{K}_{df,3}$, allowing a first comparison to the lattice spectrum.

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1. Introduction

The multitude of exotic hadrons discovered in recent decades has led to significant efforts to understand their properties directly from the underlying theory of the strong interactions, quantum chromodynamics (QCD). A noteworthy example is the doubly-charmed tetraquark, $T_{cc}(3875)^+$, observed by the LHCb collaboration in 2021 [1, 2]. This state, with quantum numbers $I(J^P) = 0(1^+)$ and minimal quark content $cc\bar{u}\bar{d}$, is a very narrow resonance decaying primarily to three-hadron states via intermediate two-hadron states, $T_{cc}^+ \rightarrow DD^* \rightarrow DD\pi$. Its structure is still a matter of debate – the proximity to the DD^* threshold suggests a meson-meson molecule, but compact tetraquark or diquark-antidiquark structures have also been suggested [3].

Lattice QCD can help clarify the properties of the T_{cc}^+ from first principles, and indeed several calculations have already investigated this state [4–9]. These works have used heavier-than-physical pion masses, for which the D^* meson is stable and the T_{cc}^+ can be studied through the simpler two-body process of DD^* scattering. The standard lattice approach to two-body scattering relies on the Lüscher formalism, which maps finite-volume lattice energies to infinite-volume scattering observables. However, in systems like DD^* , this method is limited because of the so-called left-hand cuts [10], subthreshold branch cuts in the partial-wave-projected amplitude, arising here due to u -channel single-pion exchanges between D and D^* . Such cuts invalidate the formalism when applied to lattice energies near to or on the cut, complicating the determination of the DD^* amplitude in the subthreshold region, where T_{cc}^+ poles are expected to appear.

Various proposals have been made to address the left-hand cut issue [11–16] and analyses of DD^* data taking into account the effect of the pion exchange already exist [7, 9, 12, 17]. In this talk, we employ one such proposal, i.e. the relativistic field-theoretic (RFT) three-body formalism on the isospin-0 $DD\pi$ system, as laid out in ref. [13] and first explored in ref. [17]. This method not only avoids the left-hand cut problem entirely, but naturally accommodates both DD^* and three-particle $DD\pi$ states. Since the T_{cc}^+ decays to a $DD\pi$ final state in nature, three-body states become increasingly relevant as we move towards physical pion masses. Therefore, this approach offers a flexible method to investigate the T_{cc}^+ from lattice QCD across a range of pion masses.

We start by giving a short overview of the three-body RFT formalism and how it can be used to study the T_{cc}^+ (section 2). We then describe the lattice setup and the extraction of finite-volume energies for the $DD^* + DD\pi$ system and the DD and $D\pi$ subsystems (section 3), and present the preliminary Lüscher analyses of the two-body subchannels and initial fits to the $DD^* + DD\pi$ spectrum using the three-body quantization condition (section 4).

2. Overview of the RFT approach

We briefly outline the RFT three-body formalism for the T_{cc}^+ below, following ref. [13]. In this approach, the $DD\pi$ system is split into a spectator and a pair, leading to two distinct two-particle subsystems depending on the choice of spectator flavor: $I = 1/2 D\pi$ for a D spectator, and $I = 1 DD$ for a π spectator. The key idea is then that the vector D^* meson can be encoded as a pole in the p -wave amplitude of the $D\pi$ subsystem, which allows the formalism to simultaneously handle $DD\pi$ and DD^* states. The pion mass determines whether the D^* is a bound state or resonance. An additional state to consider is the scalar $D_0^*(2300)$, which contributes to the s -wave $D\pi$ amplitude.

2.1 Finite-volume formalism

The centerpiece of the formalism is the three-particle quantization condition (QC3), which is satisfied at the finite-volume energies of the $DD^* + DD\pi$ system. For a cubic box of side L with periodic boundary conditions, it takes the form

$$\det_{ik\ell m} \left[\widehat{\mathcal{K}}_{\text{df},3}(E^*) + \widehat{F}_3(E, \mathbf{P}, L)^{-1} \right] = 0, \quad (1)$$

where (E, \mathbf{P}) are the total energy and three-momentum and $E^* = \sqrt{E^2 - \mathbf{P}^2}$ is the center-of-mass energy. The objects in the determinant are matrices in a space with indices $ik\ell m$: the first two indices, $i \in \{D, \pi\}$ and $\mathbf{k} \in \frac{2\pi}{L}\mathbb{Z}^3$, specify the flavor and momentum of the spectator particle, while ℓ and m specify the orbital angular momentum of the remaining DD or $D\pi$ pair in its rest frame. $\widehat{\mathcal{K}}_{3,\text{df}}$ is directly related to the three-particle K-matrix $\mathcal{K}_{3,\text{df}}$, a scheme-dependent infinite-volume object parameterizing the short-range three-body physics. The matrix \widehat{F}_3 is explicitly given by

$$\widehat{F}_3 \equiv \frac{\widehat{F}}{3} - \widehat{F} \frac{1}{1 + \widehat{\mathcal{M}}_{2,L}\widehat{G}} \widehat{\mathcal{M}}_{2,L}\widehat{F}, \quad \widehat{\mathcal{M}}_{2,L} \equiv \frac{1}{\widehat{\mathcal{K}}_{2,L}^{-1} + \widehat{F}}, \quad (2)$$

where \widehat{F} , \widehat{G} are kinematic functions (\widehat{F} is closely related to the multichannel Lüscher zeta function and \widehat{G} effects swaps of the spectator particle) and $\widehat{\mathcal{K}}_{2,L}$ packages information about the dynamics of the two-particle subsystems, through their K-matrices. The definitions of these objects are given in full in Section 3.2 of ref. [17]. Parameters for the two-particle K-matrices can be fixed from separate lattice or experimental values, as done in ref. [17]. In the current study, we instead extract these by direct fits to the two-body finite-volume spectra, obtained on the same ensembles as the $DD^* + DD\pi$ system.

Combining the subchannel information with the QC3, we can then constrain $\mathcal{K}_{\text{df},3}$ from the $DD^* + DD\pi$ spectrum. We use a threshold expansion to parametrize $\mathcal{K}_{\text{df},3}$ [18]:

$$\mathcal{K}_{\text{df},3}(\{\mathbf{k}'\}, \{\mathbf{k}\}) = \mathcal{K}_3^{\text{iso},1} + \mathcal{K}_3^{\text{iso},2}\Delta + \mathcal{K}_3^B\Delta_2^S + \mathcal{K}_3^E\tilde{t}_{22}, \quad (3)$$

where $\{\mathbf{k}\}, \{\mathbf{k}'\}$ are the sets of initial and final state three-momenta, respectively, and $\mathcal{K}_3^{\text{iso},1}, \mathcal{K}_3^{\text{iso},2}, \mathcal{K}_3^B$ and \mathcal{K}_3^E are real parameters. The kinematic variables Δ, Δ_2^S and \tilde{t}_{22} are defined as

$$\Delta \equiv \frac{E^2 - M^2}{M^2}, \quad \Delta_2^S \equiv \frac{\sigma'_{DD} + \sigma_{DD} - 8M_D^2}{M^2}, \quad \tilde{t}_{22} \equiv \frac{(k'_\pi - k_\pi)^2}{M^2}, \quad (4)$$

where $M \equiv 2M_D + M_\pi$ is the $DD\pi$ threshold energy, $\sigma_{DD}, \sigma'_{DD}$ are the two-body invariant masses squared for the DD subsystem in the initial and final state, while k_π, k'_π are the initial and final four-momenta of the pion.

2.2 From $\mathcal{K}_{\text{df},3}$ to the DD^* amplitude

Given the two- and three-body K-matrices, we determine the connected $DD\pi$ amplitude, \mathcal{M}_3 , by numerically solving the RFT integral equations. We use a compact matrix notation below, with multiplication signifying integration over intermediate loop momenta. All objects are projected

to definite J^P and expressed in the total orbital angular momentum and spin LS basis, so that the three-body amplitude carries channel and partial-wave indices. \mathcal{M}_3 is given by

$$\mathcal{M}_3 = \mathcal{D} + \mathcal{M}_{\text{df},3}, \quad (5)$$

where \mathcal{D} resums one-particle exchanges, and $\mathcal{M}_{\text{df},3}$ includes all short-range three-body interactions. The ladder amplitude satisfies an integral equation, which can be written schematically as

$$\mathcal{D} = -\mathcal{M}_2 \mathcal{G} \mathcal{M}_2 - \mathcal{M}_2 \mathcal{G} \mathcal{D}, \quad (6)$$

where \mathcal{G} is the single-pion exchange and \mathcal{M}_2 includes the DD and $D\pi$ amplitudes, specified through their K-matrices. The three-body K-matrix $\mathcal{K}_{\text{df},3}$ enters through the *divergence-free amplitude*, $\mathcal{M}_{\text{df},3}$, which obeys

$$\mathcal{M}_{\text{df},3} = \mathcal{L} \mathcal{T} \mathcal{R}, \quad \mathcal{T} = \mathcal{K}_{\text{df},3} - \mathcal{K}_{\text{df},3} \tilde{\rho} \mathcal{L} \mathcal{T}. \quad (7)$$

where \mathcal{L} and \mathcal{R} resum external two-body rescatterings and $\tilde{\rho}$ is a modified phase-space factor. Detailed definitions are given in section 2 of ref. [17].

The DD^* amplitude is obtained from the $DD\pi$ solution via LSZ reduction at the D^* pole of the $D\pi$ p -wave amplitude. Near the pole, the p -wave $D\pi$ amplitude is, approximately,

$$\mathcal{M}_{2, p\text{-wave}}^{(D\pi)} \sim \frac{\zeta^2}{\sigma_{D\pi} - M_{D^*}^2}, \quad (8)$$

where $\sigma_{D\pi}$ is the two-body invariant mass squared. Amputating the external pole factors of the $DD\pi$ amplitude and setting the external $D\pi$ invariant masses to the D^* mass yields the required DD^* amplitude.

3. Lattice setup and spectrum determination

This work was based on two CLS ensembles, X252 and X253; see Table 1. Both are $N_f = 2+1$ ensembles with $O(a)$ -improved Wilson fermions, with the same lattice spacing and pion mass $M_\pi \approx 280$ MeV, but different volumes. The valence charm quark mass was tuned such that the D mass is the average of the physical D^0, D^+, D_s^+ masses. The analysis presented here has primarily used the X252 ensemble, with X253 used at this stage only for the $I = 1/2$ $D\pi$ amplitude.

	L/a	T/a	$M_\pi L$	M_π [MeV]	a [fm]	N_{LapH}
X252	36	128	3.4	280	0.064	48
X253	40	128	3.7	280	0.064	64

Table 1: CLS ensembles used in this work.

The operator bases were built from local single-hadron operators projected to definite momentum and products of two or three such operators, combined to transform under the finite-volume irreps of interest. Frames with total momentum $(L\mathbf{p}/2\pi)^2 = 0, 1, 2, 3$ and individual particle momenta going up to $(L\mathbf{p}/2\pi)^2 = 3$ were considered. For $I = 1/2$ $D\pi$, the operator set includes D^*

and D_0^* operators and bilocal $D\pi$ and $D^*\pi$. For $I = 1$ DD , we have bilocal DD , D^*D , D^*D^* and $D_0^*D_0^*$ operators. Finally, for the main $DD\pi + DD^*$ system, bilocal DD^* , D^*D^* , D_0^*D , $D_0^*D^*$ and trilocal $DD\pi$ operators are used, in addition to local DD^* and D^*D^* tetraquark operators, e.g.

$$T_i^{DD^*}(t, \mathbf{p}) = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}} [\bar{u}\gamma_5 c \bar{d}\gamma_i c - \bar{d}\gamma_5 c \bar{u}\gamma_i c](t, \mathbf{x}). \quad (9)$$

The distillation framework [19] is used to compute correlator matrices. Perambulators were computed using the QUDA-LapH package [20] (number of LapH eigenvectors shown in Table 1). To compute correlators involving tetraquark operators, the position-space sampling of ref. [21] was employed. A GEVP is performed on the correlator matrix to extract the finite-volume energy levels.

For single hadrons and the D^* - and D_0^* -like levels in the $D\pi$ system, the energy extraction used single-exponential fits directly to the correlators. For all other levels, fits were performed to ratios of correlators, where the two- or three-hadron correlators are divided by the product of single-hadron correlators with momentum assignments chosen based on the overlaps to the operators, to extract energy shifts. Interacting energies are then reconstructed using continuum dispersion relations and the extracted single-hadron masses.

We emphasize the importance of tetraquark operators, which significantly impact several energy levels, as pointed out in recent works [9, 21], and note also the relevance of $DD\pi$ operators to resolve lower-lying $DD\pi$ -like states, e.g. the ground state in the rest frame A_{1u} irrep.

4. Preliminary results

This preliminary analysis consists of initial checks on each of the three relevant systems. The DD and $D\pi$ subsystems were investigated individually, with a finite-volume analysis performed on each to select reasonable K-matrix parameterizations and determine best fit parameters. The main $DD^* + DD\pi$ system was then studied, using the QC3 to fit the spectrum and determine the three-particle K-matrix, fixing the two-body parameters to those obtained from the subsystems.

4.1 Isospin-1 DD subsystem

To constrain the s -wave amplitude for the $I = 1$ DD system, eight levels were obtained on the X252 ensemble across the $A_{1g}(0)$, $A_1(1)$, $A_1(2)$ and $A_1(3)$ irreps (the number in brackets corresponds to the value of $(LP/2\pi)^2$). All eight levels lie below the D^*D^* threshold and were included in the fit. Two- and three-term effective range expansions (ERE2 and ERE3 for short) were used to parameterize the s -wave phase shift. An ERE3 parametrization of the form

$$\frac{q}{M_D} \cot \delta_{DD,s} = C_1 + C_2 \left(\frac{q}{M_D} \right)^2 + C_3 \left(\frac{q}{M_D} \right)^4, \quad (10)$$

where M_D is the D meson mass, q is the center-of-mass scattering momentum and $\delta_{DD,s}$ is the phase shift, was chosen. The best fit parameters obtained were

$$C_1 = -0.454(14), \quad C_2 = -1.96(77), \quad C_3 = -26.3(8.5), \quad (11)$$

leading to $\chi^2/\text{d.o.f.} = 5.58/5 = 1.17$. These yield a scattering length and effective range of $a_{0,s}M_D = 2.21(7)$ and $r_{0,s}M_D = -3.91(1.54)$, confirming the expectation of a relatively weak interaction. The fit results are shown in Figure 1.

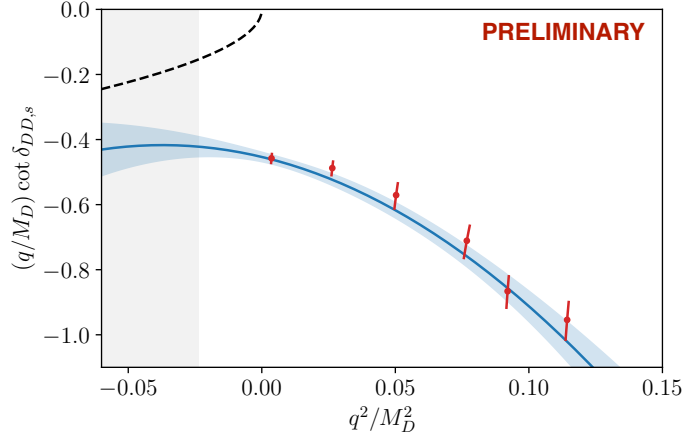


Figure 1: Plot of the s -wave phase shift for the $I = 1$ DD subsystem. The solid blue line is the best ERE3 fit described in the text and the red points show the data. Two levels included in the fit (ground states of the $(LP/2\pi)^2 = 2, 3$ frames) are not shown, as their error bars cross free-energy singularities of the zeta function. The dashed black line shows the bound state condition and the shaded gray region corresponds to the left-hand cut due to two-pion exchange.

4.2 Isospin-1/2 $D\pi$ subsystem

To constrain the s - and p -wave amplitudes for the $I = 1/2$ $D\pi$ subchannel, a total of 38 levels were extracted, 19 each on the X252 and X253 ensembles. These were obtained on the s - and p -wave dominated $A_{1g}(0)$, $T_{1u}(0)$, $A_1(1)$, $A_1(2)$ and $A_1(3)$ irreps, with the ground states in the p -wave dominated $E(1)$, $B_1(2)$, $B_2(2)$, $E(3)$ irreps also included to provide extra constraints on the D^* state. At this pion mass (~ 280 MeV), the D^* is expected to be bound, as confirmed by the spectrum shown on the left side of Figure 2.

A variety of phase shift parameterizations were used to perform the analysis. The forms chosen were modified ERE2s:

$$q \cot \delta_{D\pi,s} = \frac{1}{2} r_{0,s} (q^2 - q_{0,s}^2), \quad (12)$$

$$q^3 \cot \delta_{D\pi,p} = \frac{1}{2} r_{0,p} (q^2 - q_{0,p}^2). \quad (13)$$

For s -wave, the effective range $r_{0,s}$ and zero crossing $q_{0,s}^2$ were used as fit parameters. For p -wave, the heavy-quark effective theory (HQET) $D^*D\pi$ coupling g and the D^* mass M_{D^*} were used instead. These fix $r_{0,p}$ and $q_{0,p}^2$ uniquely by requiring that the D^* pole position and residue match between the two expressions for the p -wave amplitude, as worked out in detail in Appendix D of ref. [17].

While the D^* pole position, given by its mass M_{D^*} , seems very precisely constrained by the $D\pi$ data alone, its residue, set by the coupling g , is not. Therefore, we introduced a Bayesian prior $g = 0.55(10)$ to stabilize these preliminary fits. This value was chosen in line with ref. [17], close to the experimental value ~ 0.57 [22] and within the range of lattice-determined values $0.50 - 0.65$ [23, 24]. We observed that the coupling and its error seem to effectively be set by the prior.

For the best fit of the 32 selected levels, the parameters were

$$\begin{aligned} q_{0,s}^2/M_D^2 &= 0.0080(18), & r_{0,s}M_D &= -7.91(80), \\ M_{D^*}/M_D &= 1.07041(63), & gM_D &= 0.542(96) \end{aligned} \quad (14)$$

leading to $\chi^2/\text{d.o.f.} = 36.5/29 = 1.26$. The best fits are shown on the right-hand side of Figure 2.

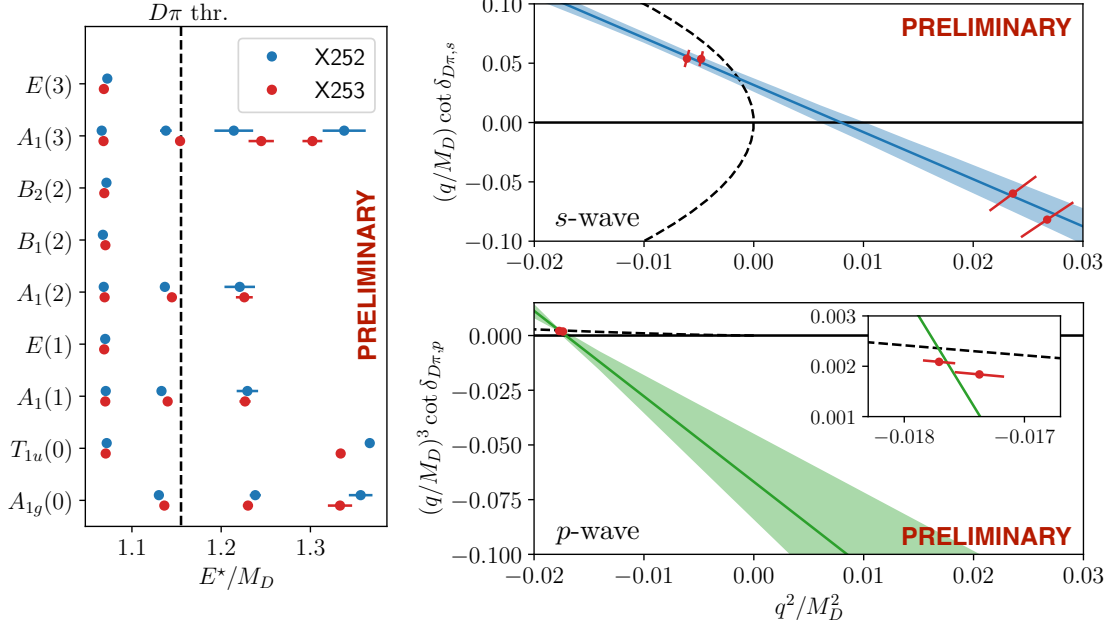


Figure 2: The $D\pi$ spectrum on each ensemble is shown on the left. On the right, the s - and p -wave phase shift plots show the best fits described in the main text. The rest frame A_{1g} and T_{1u} levels included in the fit are displayed in red. The D_0^* appears as a virtual bound state in s -wave, while the D^* is a bound state in the p -wave, as expected. The inset zooms in on the intersection of the bound state condition (dashed line).

4.3 Isospin-0 $DD^* + DD\pi$ system

We turn now to the main $DD^* + DD\pi$ system. Here, 12 levels obtained in X252 were selected for the fitting, on the $T_{1g}(0)$, $A_{1u}(0)$, $A_2(1)$ and $A_2(2)$ irreps.

A first check of the spectrum is provided by solving the QC3 of eq. (1), with the three-body K-matrix $\widehat{\mathcal{K}}_{3,\text{df}}$ set to zero and the DD and $D\pi$ two-body K-matrices entering \widehat{F}_3 fixed to the models and parameters obtained in sections 4.1 and 4.2. The plot in Figure 3 shows the comparison of the data with the QC3 prediction – although there is some correspondence between the data points and QC3 solutions, these give a rather poor description of the data with $\chi^2 \approx 247$. This is unsurprising given the observation in previous work [17] that a nonzero $\widehat{\mathcal{K}}_{3,\text{df}}$ was needed to adequately describe the lattice data of ref. [4].

In ref. [17], it was shown that the finite-volume spectrum predicted by the QC3 is most sensitive to the \mathcal{K}_3^E term in the parameterization of eq. (3). Therefore, an initial fit to the twelve levels with \mathcal{K}_3^E as the single parameter was performed. This already shows considerable improvement, lowering the χ^2 down to ~ 104 , with 11 degrees of freedom. Going further, we can include a constant $\mathcal{K}_3^{\text{iso},0}$

term, mostly impacting the A_{1u} levels and leading to a further decrease to $\chi^2 \approx 66$, with 10 degrees of freedom.

Although more terms can be added to the $\mathcal{K}_{df,3}$ parameterization, we consider that the most promising way forward is to move towards simultaneous fits to the DD , $D\pi$ and $DD^* + DD\pi$ data, as has been done for three-body systems of pions and kaons in ref. [25]. This way, the two-body subchannels can more directly inform the fit of the $DD\pi + DD^*$ data and vice-versa. This could prove helpful in resolving issues such as constraining the $D^*D\pi$ coupling g .

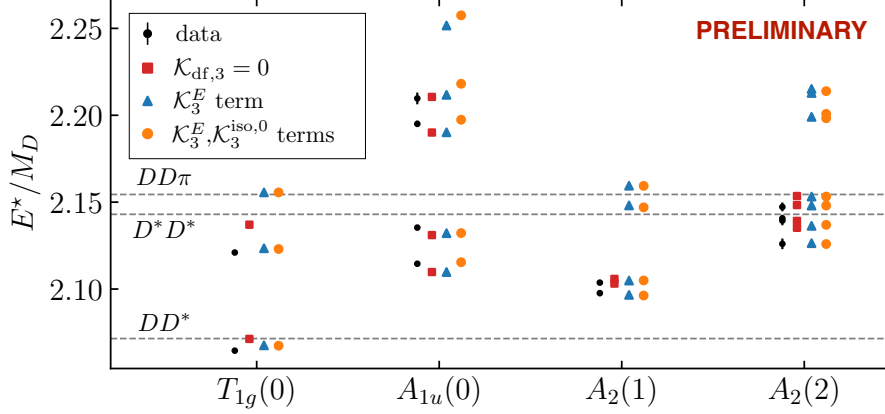


Figure 3: Comparison of the selected $DD^* + DD\pi$ levels extracted on X252 (black) with the QC3 predictions for vanishing $\mathcal{K}_{df,3}$ (red), and the two fits described in the main text (blue and orange). The relevant two- and three-body thresholds are shown as dashed lines.

5. Summary and outlook

We have presented preliminary results from a first lattice QCD study of the T_{cc}^+ tetraquark via a three-body approach [13]. This approach circumvents the left-hand cut issues that have affected previous investigations of the T_{cc}^+ via DD^* scattering using the Lüscher formalism, also treating three-body $DD\pi$ effects, which are crucial for future studies at physical or near-physical pion masses.

Finite-volume spectra for the $I = 0$ $DD^* + DD\pi$ system and the two-particle subsystems $I = 1$ DD and $I = 1/2$ $D\pi$ were extracted on the X252 CLS ensemble at $M_\pi \approx 280$ MeV, with $D\pi$ using also the X253 ensemble. The subchannel systems were analyzed individually to determine suitable models for the two-body K-matrices, required as inputs for the three-body quantization condition. The $DD^* + DD\pi$ spectrum was then fit using fixed two-body parameters and simple three-particle K-matrix parameterizations, leading to a reasonable but improvable description of the data.

In the near future, energies will be extracted on the X253 ensemble for all three systems, allowing us to probe more kinematics. The clear next step will then be to perform simultaneous fits to all three spectra, concurrently constraining the two- and three-body physics. With a reasonable determination of the three-body K-matrix, we can move on to solving the integral equations to obtain the DD^* scattering amplitude and search for poles associated with the T_{cc}^+ . The machinery for this step is largely in place from the previous work of ref. [17].

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References

- [1] LHCb collaboration, *Observation of an exotic narrow doubly charmed tetraquark*, *Nature Phys.* **18** (2022) 751 [2109.01038].
- [2] LHCb collaboration, *Study of the doubly charmed tetraquark T_{cc}^+* , *Nature Commun.* **13** (2022) 3351 [2109.01056].
- [3] C. Hanhart, *Hadronic molecules and multiquark states*, 2504.06043.
- [4] M. Padmanath and S. Prelovsek, *Signature of a Doubly Charm Tetraquark Pole in DD^* Scattering on the Lattice*, *Phys. Rev. Lett.* **129** (2022) 032002 [2202.10110].
- [5] S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun et al., *$T_{cc}^+(3875)$ relevant DD^* scattering from $N_f = 2$ lattice QCD*, *Phys. Lett. B* **833** (2022) 137391 [2206.06185].
- [6] Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda and J. Meng, *Doubly Charmed Tetraquark T_{cc}^+ from Lattice QCD near Physical Point*, *Phys. Rev. Lett.* **131** (2023) 161901 [2302.04505].
- [7] S. Collins, A. Nefediev, M. Padmanath and S. Prelovsek, *Toward the quark mass dependence of T_{cc}^+ from lattice QCD*, *Phys. Rev. D* **109** (2024) 094509 [2402.14715].
- [8] HADRON SPECTRUM collaboration, *Near-threshold states in coupled $DD^*-D^*D^*$ scattering from lattice QCD*, *Phys. Rev. D* **111** (2025) 034511 [2405.15741].
- [9] S. Prelovsek, E. Ortiz-Pacheco, S. Collins, L. Leskovec, M. Padmanath and I. Vujmilovic, *Doubly heavy tetraquarks from lattice QCD: Incorporating diquark-antidiquark operators and the left-hand cut*, *Phys. Rev. D* **112** (2025) 014507 [2504.03473].

- [10] M.-L. Du, A. Filin, V. Baru, X.-K. Dong, E. Epelbaum, F.-K. Guo et al., *Role of Left-Hand Cut Contributions on Pole Extractions from Lattice Data: Case Study for $T_{cc}(3875)^+$* , *Phys. Rev. Lett.* **131** (2023) 131903 [2303.09441].
- [11] A.B. Raposo and M.T. Hansen, *Finite-volume scattering on the left-hand cut*, *JHEP* **08** (2024) 075 [2311.18793].
- [12] L. Meng, V. Baru, E. Epelbaum, A.A. Filin and A.M. Gasparyan, *Solving the left-hand cut problem in lattice QCD: $T_{cc}(3875)^+$ from finite volume energy levels*, *Phys. Rev. D* **109** (2024) L071506 [2312.01930].
- [13] M.T. Hansen, F. Romero-López and S.R. Sharpe, *Incorporating $DD\pi$ effects and left-hand cuts in lattice QCD studies of the $T_{cc}(3875)^+$* , *JHEP* **06** (2024) 051 [2401.06609].
- [14] R. Bubna, H.-W. Hammer, F. Müller, J.-Y. Pang, A. Rusetsky and J.-J. Wu, *Lüscher equation with long-range forces*, *JHEP* **05** (2024) 168 [2402.12985].
- [15] S.M. Dawid, A.W. Jackura and A.P. Szczepaniak, *Finite-volume quantization condition from the N/D representation*, *Phys. Lett. B* **864** (2025) 139442 [2411.15730].
- [16] A.B. Raposo, R.A. Briceño, M.T. Hansen and A.W. Jackura, *Extracting scattering amplitudes for arbitrary two-particle systems with one-particle left-hand cuts via lattice QCD*, *JHEP* **06** (2025) 186 [2502.19375].
- [17] S.M. Dawid, F. Romero-López and S.R. Sharpe, *Finite- and infinite-volume study of $DD\pi$ scattering*, *JHEP* **01** (2025) 060 [2409.17059].
- [18] T.D. Blanton and S.R. Sharpe, *Three-particle finite-volume formalism for $\pi^+\pi^+K^+$ and related systems*, *Phys. Rev. D* **104** (2021) 034509 [2105.12094].
- [19] HADRON SPECTRUM collaboration, *A Novel quark-field creation operator construction for hadronic physics in lattice QCD*, *Phys. Rev. D* **80** (2009) 054506 [0905.2160].
- [20] Cosmon Collaboration, *quda laph: Lattice QCD computations with stochastic Laph and distillation using the QUDA library*, 2025.
- [21] A. Stump and J.R. Green, *Position-space sampling for local multiquark operators in lattice QCD using distillation and the importance of tetraquark operators for $T_{cc}(3875)^+$* , *Phys. Rev. D* (2026) (to be published) [2510.26459].
- [22] BABAR collaboration, *Measurement of the $D^*(2010)^+$ natural line width and the $D^*(2010)^+ - D^0$ mass difference*, *Phys. Rev. D* **88** (2013) 052003 [1304.5009].
- [23] D. Becirevic and F. Sanfilippo, *Theoretical estimate of the $D^* \rightarrow D\pi$ decay rate*, *Phys. Lett. B* **721** (2013) 94 [1210.5410].
- [24] K.U. Can, G. Erkol, M. Oka, A. Ozpineci and T.T. Takahashi, *Vector and axial-vector couplings of D and D^* mesons in 2+1 flavor Lattice QCD*, *Phys. Lett. B* **719** (2013) 103 [1210.0869].
- [25] Z.T. Draper, A.D. Hanlon, B. Hörz, C. Morningstar, F. Romero-López and S.R. Sharpe, *Interactions of πK , $\pi\pi K$ and $KK\pi$ systems at maximal isospin from lattice QCD*, *JHEP* **05** (2023) 137 [2302.13587].
- [26] Jülich Supercomputing Centre, *JUWELS Cluster and Booster: Exascale pathfinder with modular supercomputing architecture at Jülich Supercomputing Centre*, *J. Large-Scale Res. Facil.* **7** (2021) A183.