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Understanding the spin-spin interactions among heavy quarks at finite temperature

Swagatam Tah,[a,*] **Dibyendu Bala,**[b] **Olaf Kaczmarek**[b] **and Sayantan Sharma**[a]

[a] *The Institute of Mathematical Sciences, a CI of Homi Bhabha National Institute, Chennai, 600113, India*
[b] *Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*

E-mail: swagatamt@imsc.res.in

In this proceeding, we report the non-perturbative potential between a heavy quarkantiquark pair in a QCD plasma at finite temperature. We compute the spin-dependent component of the thermal potential using temporal Wilson line correlators with insertions of the color-magnetic field. The calculations are performed in quenched QCD at 1.5 times the deconfinement temperature T_d, on a four-dimensional lattice with spatial extent $N_s = 68$, and temporal extents $N_\tau = 16$ and 20. For the first time, we show that $V_{\mathrm{ss}}(r)$ develops an imaginary part at finite temperature, a feature similar to that observed in the static potential. Finally, we discuss the physical implications of our results for understanding the dissociation of quarkonium bound states in the quarkgluon plasma.

The 42nd International Symposium on Lattice Field Theory (LATTICE2025)
2-8 November 2025
Tata Institute of Fundamental Research, Mumbai, India

*Speaker

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ISSN 1824-8039. Published by SISSA Medialab.

1. Introduction and Motivation

Quantum Chromodynamics (QCD) describes the strong interactions between quarks and gluons [1, 2]. At finite temperature, this theory exhibits either a first-order or a crossover-like phase transition from a color-singlet hadronic phase to a quarkgluon plasma (QGP) phase consisting of color degrees of freedom, in the absence [3] and presence [4] of dynamical fermions, respectively. At the Relativistic Heavy Ion Collider as well as in Large Hadron Collider experiments, signatures of the QGP phase have been observed most convincingly in the most central heavy-ion collisions (HIC) [5, 6]. In HIC, the evolution from the QGP phase to hadrons occurs rapidly within a timescale ~ 10 fm, making its direct detection nearly impossible. Nevertheless, several probes are used to study the properties of the QGP, such as jet quenching [7], collective flow of the hadrons [8], suppression of quarkonium states [9], and strangeness enhancement[10]. Quarkonia are bound states of a heavy quarkantiquark pair, each with mass $M \gg \Lambda_{\rm QCD}$, $\Lambda_{\rm QCD}$ being the strong interaction scale. Heavy quarks are produced at very early stages of the HIC ($t \ll 1$ fm) and evolve throughout the lifetime of the fireball. As quarkonia interact with the QGP during this evolution, information about medium properties get encoded in their spectral functions. Therefore, these serve as excellent probes for understanding the properties of the strongly coupled QGP.

The heavy quarks for most practical purposes have been treated non-relativistically, since the heavy-quark mass $M \gg \Lambda_{\rm QCD}$. In finite temperatures, $M \gg T$ for bottom quarks, however for the charm this condition might not be well satisfied. The quarkonium correlation function can be formulated in terms of a non-relativistic Schrödinger equation with an inter-quark potential [11]. One can improve the applicability of this so-called NRQCD formalism, by calculating the inter-quark potential beyond the non-relativistic *infinite* quark mass limit in powers of the inverse heavy-quark mass [12, 13]. The spin-dependent term, which arises at $O(1/M^2)$ is a quantum-mechanical correction to the non-relativistic potential lifting the degeneracy between the pseudo-scalar and vector quarkonium states. At zero temperature, the static potential exracted from lattice QCD studies can be well described by the Cornell potential, with a string-like term dominating at long distances and a Coulomb-like term that describes the interactions at short distances [14]. The spin-dependent potential has also been extracted from lattice QCD at zero temperatures which turns out to be short-ranged consistent with the expectation from perturbation theory [15, 16]. This potential has been used extensively to explain the hyperfine splittings between quarkonium states with different quantum numbers at zero temperature.

At finite temperatures, the static potential was calculated for the first time using hard thermal loop (HTL) perturbation theory, where it was found to acquire an imaginary part [17]. This feature is also supported by non-perturbative lattice studies [18]. At temperatures above the deconfinement temperature, the real part of the static potential can be described by a Debye-screened potential, while the imaginary part is related to Landau damping, arising from the scattering of the heavy-quark pair with in-medium hard partons. Later, within weak-coupling pNRQCD, it was shown that the imaginary part also receives contributions from gluodissociation, wherein a quarkonium state absorbs a thermal gluon transitioning to a color-octet state, and subsequently dissociates [19]. The imaginary part of the inter-quark potential is also responsible for the finite decay width of the in-medium quarkonium spectral function. If the magnitude of the imaginary part becomes significantly larger than the real part of the static potential, a bound-state formation is no longer

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possible leading to dissociation in the QGP. Charmonium state J/ψ is expected to dissociate around $1.5\,T_d$ known from quenched QCD [20, 21], and do show appreciable thermal broadening around 300 MeV [18].

Given these observations, it is natural to ask whether the spin-dependent potential also develops an imaginary component at finite temperatures. If such a contribution exists, quarkonium states with different quantum numbers would receive different corrections due to this $O(1/M^2)$ term of the inter-quark potential. This would imply that pseudo-scalar and vector quarkonia might acquire different thermal decay widths in the medium. In particular, for charmonium states, this correction term is expected to be significant, which in turn will help in constructing more realistic spectral functions from lattice calculations. A realistic reconstruction of the charmonium spectral functions will improve the estimates for the dilepton production rates as a function of temperature. In this proceeding we discuss our formalism and first results on the spin-spin interactions between heavy quarks. To our knowledge, a first-principles study of spin interactions in the context of quarkonium states at finite temperatures has not yet been carried out. In this work, we address this from a non-perturbative lattice study and comparing our results with the leading-order perturbative HTL calculations.

2. Theoretical framework

At zero temperature, the static potential can be extracted on the lattice from the exponential fall-off of a gauge-invariant Wilson loop of temporal extent τ and spatial separation r in the limit of τ being infinitely large. However, the extraction of the thermal potential is considerably more challenging due to the emergence of an imaginary part. Moreover, the τ direction is compact and limited to a finite size given by the inverse of temperature hence it is not possible to access the $\tau \to \infty$ limit. The thermal potential is however well defined in Minkowski time, whereas in lattice QCD we can calculate the Euclidean correlator between the quark-antiquark pair. This implies that the extraction of the thermal potential requires an analytic continuation from Euclidean to Minkowski time. However, analytic continuation is an ill-posed problem, as only a finite set of data points with statistical uncertainties is available from lattice calculations, allowing for infinitely many possible solutions. Therefore, additional physics-motivated inputs are required to constrain the space of solutions. Various approaches have been developed to address this problem, including maximum entropy method, Bayesian techniques, and other physics-driven analytic continuation strategies, see for e.g. the Lattice plenary on this topic [22]. These studies consistently indicate the presence of sizable non-perturbative effects in the thermal static potential.

In order to derive the potential energy between a $q\bar{q}$ pair with spatial separation $r = |\vec{x} - \vec{y}|$ within the NRQCD framework, we consider the operator that creates a heavy quarkantiquark pair in a given quantum channel Γ, defined as

$$\hat{\mathcal{P}}^\dagger(r,\tau) = \chi^\dagger(x)\,\Gamma\,U(\vec{x}-\vec{y},\tau)\,\theta(y)\,.$$

Here, $U(\vec{x}-\vec{y},\tau)$ is the gauge link connecting the $q\bar{q}$ pair, which ensures gauge invariance. The corresponding correlation function is denoted

$$C_\Gamma(r,\tau) = \langle P^\dagger(r,\tau)\,P(r,0)\rangle_T\,,$$

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where the expectation value is taken over the thermal Gibbs ensemble. Using the propagators for non-interacting heavy quarks, one obtains

$$C_\Gamma(r,\tau) = \exp(-2\,M\,\tau)\,\mathrm{Tr}_c[\Gamma^2]\,W_T(r,\tau)\left[1 + \frac{1}{4\,M^2}\left(W^{\mathrm{int}}_{\mathrm{BB}}(r,\tau) + W^{\mathrm{self}}_{\mathrm{BB}}(r,\tau)\right)\right]. \qquad (1)$$

Here, $W_T(r,\tau) \equiv \langle \mathrm{Tr}_c[W(r,\tau)]\rangle_T$ denotes the thermal expectation value of the gauge-invariant Wilson loop. The term $W^{\mathrm{int}}_{\mathrm{BB}}$ represents the interaction between the quarkantiquark pair mediated by the chromo-magnetic field B, whereas $W^{\mathrm{self}}_{\mathrm{BB}}$ corresponds to the self-energy contribution of the quark or antiquark subject to the same color magnetic field. These contributions are given by the following expressions,

$$W^{\mathrm{int}}_{\mathrm{BB}}(r,\tau) = \mathcal{X}_{ij}\int_0^\tau\int_0^\tau d\tau'\,d\tau''\,\frac{\left\langle \mathrm{Tr}_c\left[\mathcal{T}\,W(r,\tau)\,g_0 B_i(\vec{y},\tau')\,g_0 B_j(\vec{x},\tau'')\right]\right\rangle_T}{W_T(r,\tau)}, \qquad (2)$$

$$W^{\mathrm{self}}_{\mathrm{BB}}(r,\tau) = \mathcal{Y}_{ij}\int_0^\tau\int_0^\tau d\tau'\,d\tau''\,\frac{\left\langle \mathrm{Tr}_c\left[\mathcal{T}\,W(r,\tau)\,g_0 B_i(\vec{x},\tau')\,g_0 B_j(\vec{x},\tau'')\right]\right\rangle_T}{W_T(r,\tau)}, \qquad (3)$$

where $\mathcal{T}$ denotes time ordering of the operators. Here the coefficients $\mathcal{X}_{ij}$ and $\mathcal{Y}_{ij}$ are defined as

$$\mathcal{X}_{ij} = \frac{\mathrm{Tr}_d\left(\Gamma\sigma_i\Gamma\sigma_j\right)}{\mathrm{Tr}_d\left(\Gamma^2\right)}, \qquad \mathcal{Y}_{ij} = \frac{\mathrm{Tr}_d\left(\sigma_i\sigma_j\Gamma\Gamma\right)}{\mathrm{Tr}_d\left(\Gamma^2\right)}.$$

The thermal potential is defined in real time through analytic continuation from Euclidean time

$$V_\Gamma(r) = \lim_{t\to\infty} i\partial_t\ \log C_\Gamma(r,\tau\to it) = 2\,M + V_{\mathrm{static}}(r) + \frac{V^\Gamma_{\mathrm{ss}}(r)}{4\,M^2}. \qquad (4)$$

Here, V_{static} is the standard static potential defined in terms of the Wilson loop,

$$V_{\mathrm{static}}(r) = \lim_{t\to\infty} i\partial_t\ \left[\log W_T(r,\tau\to it)\right], \qquad (5)$$

whereas the spin-spin interaction potential is denoted by

$$V^\Gamma_{\mathrm{ss}}(r) = \lim_{t\to\infty} i\partial_t\ \left[W^{\mathrm{int}}_{\mathrm{BB}}(r,\tau\to it) + W^{\mathrm{self}}_{\mathrm{BB}}(r,\tau\to it)\right]. \qquad (6)$$

Note that for the pseudoscalar channel, $\Gamma = 1$, while for the vector channel, $\Gamma = \sigma_k$ with $k = 1,2,3$. This leads to $\mathcal{X}_{ij} = \delta_{ij}$ in the pseudoscalar case and $\mathcal{X}_{ij} = -\delta_{ij}/3$ in the vector channel, while $\mathcal{Y}_{ij} = \delta_{ij}$ irrespective of the quantum channel. Our ultimate aim is to calculate Eq. (4) non-perturbatively on the lattice. In the next section, we describe how these correlators are implemented within the lattice framework. We have also performed a leading-order perturbative computation of color-magnetic-fieldinserted correlators in Euclidean space using the HTL-resummed propagator, and subsequently extract them in real time. A similar analysis for the static potential in color-singlet and color-octet channels has been carried out previously in Refs. [18, 23, 24]. Using analogous techniques, the total spinspin potential can be written as

$$V^\Gamma_{ss}(r) = -2\,C_F g_0^2\int\frac{d^3\vec{p}}{(2\pi)^3}\left(1 + \mathcal{X}e^{-i\vec{p}\cdot\vec{r}}\right)|\vec{p}|^2\left[\frac{1}{|\vec{p}|^2 + \Pi_T(0,\vec{p})} + 2\,i\,T\lim_{p_0\to 0}\frac{\rho_T(p_0,\vec{p})}{p_0}\right]. \qquad (7)$$

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Similar to the static potential, the spinspin interaction also develops an imaginary part at finite temperatures which will lift the degeneracy between pseudo-scalar and vector quarkonium states. The Euclidean spin correlator introduced in the previous section, behaves as

$$W^{\rm int}_{\rm BB}(r,\tau) \approx \tau\, f(r) - g(r)\, \ln\left[\sin\left(\frac{\pi\tau}{\beta}\right)\right] + \cdots\,, \tag{8}$$

which upon analytic continuation to real-time and subsequent time derivation gives rise to

$$\lim_{t\to\infty} i\partial_t W^{\rm int}_{\rm BB}(r,t) \approx \Re V^{\rm int}_{\rm BB}(r) - i\,\Im V^{\rm int}_{\rm BB}(r)\,. \tag{9}$$

where $\Re V^{\rm int}_{\rm BB}(r) \equiv -f(r)$ and $\Im V^{\rm int}_{\rm BB}(r) \equiv \frac{\pi}{\beta} g(r)$ denote the real and imaginary parts of the thermal spin potential, respectively. To extract the real and imaginary parts of the spin-interaction potential from Euclidean lattice correlators, we will utilize the leading-order perturbative structure in Eq. (8).

3. Lattice implementation

We calculate the expecation values of the operators on the vacuum states denoted by $W_T(r,\tau)$, $W^{\rm int}_{\rm BB}(r,\tau)$, and $W^{\rm self}_{\rm BB}(r,\tau)$ on the lattice. All these observables are gauge invariant but suffer from large statistical noise. Achieving a reliable signal therefore requires very high statistics, making the computation computationally expensive. To address this, we instead calculate the temporal Wilson-line correlator, which consists of two Wilson lines of length τ separated by a spatial distance r. Such an correlator is defined for a gauge choice which is local along the temporal direction [25, 26]. At zero temperature, the gauge-invariant Wilson loop and this gauge-fixed Wilson-line correlator leads to the same static potential extracted from the long-time behavoir of both these quantities. This equivalence has been used to extract the zero-temperature static potential in Refs. [27, 28].

At leading order (LO) HTL perturbation theory, both these observables lead to the same imaginary part of the potential, thereby validating this approach [29]. For the color-magnetic-fieldinserted correlators, the leading-order contributions are also identical for both operators. We implement the gauge-fixing procedure following Ref. [30]. For the color-magnetic field operator, we use the clover discretization, which has reduced cutoff effects compared to the usual plaquette definition. To further reduce statistical noise and suppress ultraviolet fluctuations, we employ Zeuthen flow [31], which removes $O(a^2)$ cutoff effects and smoothens the gauge field fluctuations at $r < \sqrt{8\tau_F}$, where τ_F denotes the flow time.

We perform our calculations on quenched SU(3) gauge configurations generated using the standard Wilson gauge action, with heat-bath updates supplemented by four over-relaxation steps per update. The configurations are generated at a single temperature $T \approx 1.5\,T_d$ where T_d is the deconfinement transition temperature. The spatial extent of the lattice is fixed at $N_s = 68$ and the temporal extent are varied $N_\tau = 16, 20$ to study the approach the continuum limit. To determine the lattice spacing, we use the Sommer scale with $r_0\Lambda_{\overline{MS}} = 0.62(1)$ and $r_0 T_d = 0.7457(45)$ [32], which corresponds to $r_0 = 0.472(8)$ fm. In our simulations we fix the flow time accordingly for different lattice spacings such that τ_F remains constant in physical units. This allows for a controlled zero flow-time extrapolation after performing the continuum extrapolation.

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N_s	N_τ	β	a (fm)	$N_\#$	τ_F/a^2
68	16	6.870	0.0264	1000	0.100, 0.200, 0.300, 0.600
68	20	7.049	0.0211	1000	0.1564, 0.3132, 0.4697, 0.9380

Table 1: Details of our lattice simulations for quenched SU(3) gauge theory for $T/T_d \sim 1.5$.

4. Results

Extracting the color-magnetic fieldinserted correlators is computationally very demanding, as it requires calculating $(\tau + 1)^2$ correlators for each value of τ. A physically meaningful result is obtained only after taking the continuum limit and subsequently removing the dependence on the flow time. The Wilson-line correlators with color-magnetic field insertions has non-trivial renormalization [33], which depends on the chosen regularization scheme. In this case, renormalization of the theory parameters alone is not sufficient. In order to obtain a physical correlator that is independent of the regularization scheme, one must introduce a flow-time dependent renormalization coefficient that removes the logarithmic divergence arising at small flow time. This procedure was first applied in the context of heavy quark diffusion coefficient calculations [34, 35]. A proper determination of the renormalization factor would require a full next-to-leading order (NLO) calculation of these correlators, which is currently not available in the literature. Nevertheless, we expect that the renormalization effects do not significantly alter our qualitative conclusions. We outline below our strategy for extracting the thermal potential:

1. Extract the real and imaginary parts of the static and spin-dependent potentials from Euclidean lattice correlators in Eqs. (3, 2) using the fit ansatz given in Eq. 8.

2. Perform a continuum extrapolation for these potentials at a fixed flow time.

3. Renormalize the spin-dependent potentials using the renormalization factor provided in Ref. [33, 34] at each flow time, followed by a zero flow-time extrapolation.

The spin potential presented here is accurate up to an overall multiplicative renormalization factor, which will be fixed and reported in our upcoming publication [36]. Before taking the continuum limit of the spin-dependent potential, we subtract the leading-order divergence of the self-spin contribution, $V_{\rm ss}^{\rm self}(0)$, in order to obtain a well-defined continuum limit. Without this subtraction, a controlled linear extrapolation to zero flow time is not possible. In all plots, we present results starting from $r = 0.05$ fm to be well-beyond the typical lattice spacings.

Our results for the real-part for the spin-dependent potentials for charm and bottom states are compiled in Fig. 1 which indicates that these are short-ranged, consistent with expectations from zero temperature lattice perturbation theory. The real part of the spin-potential can be parametrized as lattice regularized Dirac delta function. The charm and bottom quark masses are chosen to be $M_c \sim 1.35$ GeV and $M_b \sim 4.78$ GeV, respectively [18]. In Fig. 2, the results for the imaginary part of the spin-dependent potential is shown which remains finite at short distances for both the pseudoscalar and vector channels, and approaches a constant value for $r \gtrsim 0.2$ fm. Another important observation is that, at short distances, the total thermal potential is dominated by the

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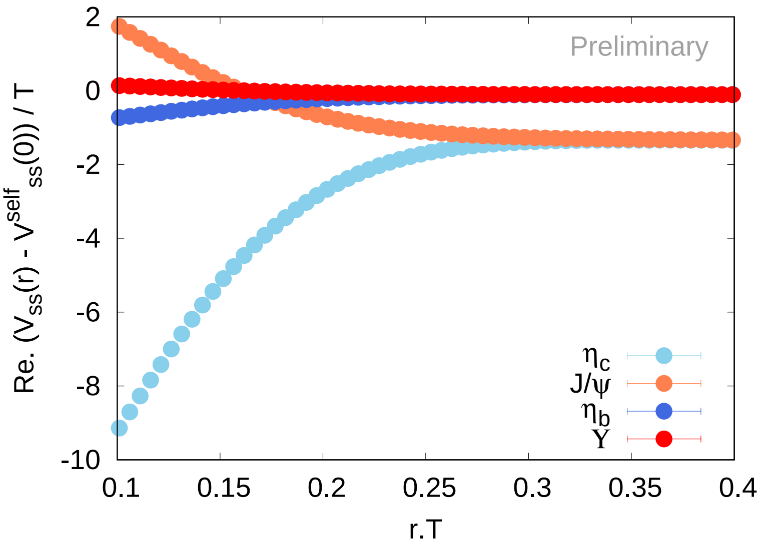

Figure 1: Real part of the total spin-dependent interaction potential between a heavy $q\bar{q}$ pair at $T \sim 1.5\,T_d$, shown for both charmonium and bottomonium states in the pseudoscalar and vector channels. The behaviour is similar to the zero temperature spin potential which is a lattice regularized Dirac delta function.

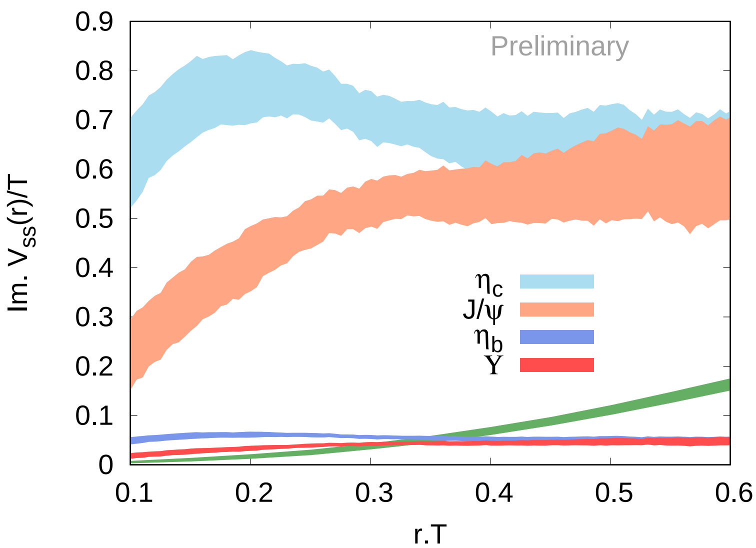

Figure 2: Imaginary part of the total spin-dependent interaction potential between a heavy $q\bar{q}$ pair at $T \sim 1.5\,T_d$, shown for both charmonium and bottomonium states in the pseudoscalar and vector channels. This potential is presented up-to an overall multiplicative renormalization factor. Imaginary part of the static potential is also as a green band as comparison.

spin-dependent contribution. This strongly indicates the necessity of including spin-dependent interactions in the construction of charmonium spectral functions.

Furthermore, the difference in the imaginary part of the spin-dependent potential between the pseudoscalar and vector channels at short distances suggests distinct thermal decay widths, which are crucial for understanding their respective dissociation temperatures. A quantitative

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determination of this effect requires computing spin-dependent potentials at multiple temperatures and constructing the corresponding quarkonium spectral functions in both channels. Qualitatively, we expect the thermal decay width of the pseudoscalar to be larger than that of the vector state, as it recieves larger contribution from the imaginary part of the spin-dependent potential.

5. Summary

For the first time, we demonstrate that the spin-dependent potential acquires an imaginary component at finite temperature both from a non-perturbative lattice study in quenched QCD and supported by our calculation in leading order HTL perturbation theory. The details will be reported in our forthcoming publication [36]. At short distances, the thermal potential is found to be dominated by the spin-dependent contribution when compared to the static potential. This highlights the importance of including spin-dependent interactions when reconstructing the quarkonium spectral functions. Another important observation is that the imaginary part of the spin-dependent potential is larger in the pseudoscalar than in the vector channel at short distances. This qualitatively indicates a larger thermal decay width for the pseudoscalar state. A quantitative understanding of this trend requires the reconstruction of the spectral functions for both pseudoscalar and vector quarkonia using the corresponding spin-dependent potentials.

6. Acknowledgements

This work was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) project number 315477589 TRR 211. All of the computations have been performed on the GPU Cluster at Bielefeld University using the `SIMULATeQCD` library [37]. S. T. and D. B. acknowledge helpful discussions with Saumen Datta during the Lattice conference.

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