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The inner structure of the $X(3872)$ via radiative decays

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The nature of the $X(3872)$ remains one of the central open problems in heavy-hadron spectroscopy. In this contribution, we study the radiative decays $X(3872) \to J/\psi\,\gamma$ and $X(3872) \to \psi'\gamma$ within a compact tetraquark interpretation. We employ a non-relativistic framework in which the decay amplitude is determined by the initial-state tetraquark wave function. This wave function is constructed using the Born–Oppenheimer approximation, treating the heavy charm quarks as slow degrees of freedom and the light quarks as fast ones. The resulting Born–Oppenheimer potential is obtained variationally and used to solve the effective Schrödinger equation for the $c\bar{c}$ pair. We compute the ratio of radiative branching fractions and obtain $\mathcal{R}_{\text{th}} = 1.4 \pm 0.3$, in good agreement with the recent LHCb measurement. Beyond this observable, the same framework provides a systematic description of the spectrum of compact $c\bar{c}q\bar{q}$ states, highlighting the versatility of the Born–Oppenheimer approach in the study of exotic hadrons.

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1. Introduction

Since its discovery in 2003, the $X(3872)$ has remained one of the most intriguing states in the heavy-quarkonium sector. It is a very narrow $J^{PC} = 1^{++}$ resonance with a mass extremely close to the $D^0\bar{D}^{*0}$ threshold [1]. Its dominant decay into $D^0\bar{D}^0\pi^0$ naturally suggests a molecular interpretation as a weakly bound $D^0\bar{D}^{*0}$ system with a binding energy of order $O(\text{keV})$ [2], corresponding to a shallow bound state in the language of low-energy scattering theory [3].

Despite this appealing picture, the sizable prompt production cross section of the $X(3872)$ at large transverse momentum, $p_T \gtrsim 15$ GeV, observed in proton–proton collisions [4], is difficult to reconcile with a purely molecular scenario [5]. As a result, the internal structure of the $X(3872)$ remains an open question, with several competing interpretations proposed in the literature, including conventional charmonium states [6], compact tetraquarks [7, 8], and open-charm molecular configurations [9] (see Ref. [10] for a review).

Further insight is provided by the radiative decays $X \to \psi'\gamma$ and $X \to J/\psi\,\gamma$. The LHCb collaboration has recently measured the ratio [11]

$$\mathcal{R}_{\text{exp}} = \frac{\text{Br}(X \to \psi'\gamma)}{\text{Br}(X \to J/\psi\,\gamma)} = 1.67 \pm 0.21 \pm 0.12 \pm 0.04\,, \tag{1}$$

which provides a sensitive probe of the internal structure of the state. In particular, calculations based solely on the universal properties of shallow bound states typically predict values of $\mathcal{R}$ well below unity [7].

In this work, we briefly review the non-relativistic framework used to compute $\mathcal{R}$ within the compact tetraquark hypothesis, following Refs. [7, 12]. The calculation requires detailed knowledge of the tetraquark wave function, which we obtain by modeling the compact $c\bar{c}q\bar{q}$ system within the Born–Oppenheimer approximation.

2. Radiative decays of the $X(3872)$

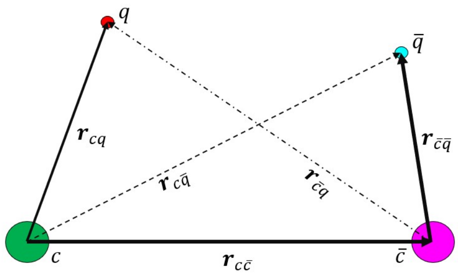

Figure 1: Coordinate system adopted.

At lowest order, the process $X \to \psi^{(\prime)}\gamma$ is dominated by the annihilation of the $q\bar{q}$ pair. Without loss of generality, we assume that the annihilation takes place at the origin of the reference frame. Defining $\psi(\boldsymbol{r}_{c\bar{c}})$ as the wave function of the final charmonium state, which depends on the relative distance $r_{c\bar{c}}$ (see Fig. 1 for the notation) between the $c\bar{c}$ pair, the transition amplitude A in the X rest frame, at fixed photon three-momentum $\boldsymbol{k}$, is given by[1]

$$A\left(X \to \psi^{(\prime)}\gamma\right) = \mathcal{F} \int_{\boldsymbol{r}_{c\bar{c}},\boldsymbol{r}_{cq}} e^{-i\boldsymbol{k}\cdot\left(\frac{\boldsymbol{r}_{c\bar{c}}}{2}-\boldsymbol{r}_{cq}\right)}\, \psi(|\boldsymbol{r}_{c\bar{c}}|)\, \Psi_{c\bar{c}q\bar{q}}(\boldsymbol{r}_{c\bar{c}},\boldsymbol{r}_{cq})\,, \tag{2}$$

The factor $\mathcal{F}$ collects common contributions that cancel out in the calculation of the ratio $\mathcal{R}$. The X wave function is assumed to factorize as

$$\Psi_{c\bar{c}q\bar{q}}(\boldsymbol{r}_{c\bar{c}},\boldsymbol{r}_{cq}) = \Psi_{c\bar{c}}(|\boldsymbol{r}_{c\bar{c}}|)\, \Psi_{q\bar{q}}(|\boldsymbol{r}_{cq}|, |\boldsymbol{r}_{cq} - \boldsymbol{r}_{c\bar{c}}|)\,. \tag{3}$$

The scale separation exploited by is the separation between the mass m_Q of the heavy quarks and the energy scale of the gluons or light quarks part of the binding, which is of the order of the non-perturbative hadronic scale $\Lambda_{\rm QCD}$. The value of $\mathcal{R}$ depends on the ratio of the squared moduli of the amplitudes in Eq. (2), on the ratio of phase spaces $\Phi = 0.26$, and on the polarization factor $\mathcal{P} = 0.98$. These contributions do not cancel in the ratio, since they depend on the momentum $\boldsymbol{k}$ of the emitted photon,

$$|\boldsymbol{k}| = \frac{M_X^2 - M_{\psi^{(\prime)}}^2}{2M_X}\,. \tag{4}$$

The exponential factor comes from the recoil of the $c\bar{c}$ pair against the photon with non-relativistic velocity $\boldsymbol{V}$ [10]. Putting everything together, one obtains

$$\mathcal{R}_{\rm th} = \Phi\,\mathcal{P} \left|\frac{A\,(X \to \psi'\gamma)}{A\,(X \to \psi\gamma)}\right|^2 \simeq 0.25 \left|\frac{A\,(X \to \psi'\gamma)}{A\,(X \to \psi\gamma)}\right|^2\,. \tag{5}$$

In order to complete the calculation and compare the result with the LHCb measurment [11] in Eq. (1), we need to determine the wave function $\Psi_{c\bar{c}q\bar{q}}$. In the compact scenario, this can be achieved, as we will show in the next section, by employing the Born–Oppenheimer (BO) approximation.

3. Born-Oppenheimer approximation

Within the BO approximation, the tetraquark wave function $\Psi_{c\bar{c}q\bar{q}}$ can be factorized into a product of a wave function describing the fast degrees of freedom (the light quarks), $\Psi_{q\bar{q}}$, and a wave function describing the slow degrees of freedom (the heavy quarks), $\Psi_{c\bar{c}}$. Using the coordinate notation in Fig. 1, this factorization reads

$$\Psi_{c\bar{c}q\bar{q}}(\boldsymbol{r}_{cq},\boldsymbol{r}_{\bar{c}\bar{q}},\boldsymbol{r}_{c\bar{c}}) = \Psi_{c\bar{c}}(\boldsymbol{r}_{c\bar{c}})\, \Psi_{q\bar{q}}(\boldsymbol{r}_{cq},\boldsymbol{r}_{\bar{c}\bar{q}};\boldsymbol{r}_{c\bar{c}})\,. \tag{6}$$

The light-quark wave function $\Psi_{q\bar{q}}$ is obtained as the solution of the Schrödinger equation

$$H_{q\bar{q}}\,\Psi_{q\bar{q}}(\boldsymbol{r}_{cq},\boldsymbol{r}_{\bar{c}\bar{q}};\boldsymbol{r}_{c\bar{c}}) = \Delta E(r_{c\bar{c}})\,\Psi_{q\bar{q}}(\boldsymbol{r}_{cq},\boldsymbol{r}_{\bar{c}\bar{q}};\boldsymbol{r}_{c\bar{c}})\,, \tag{7}$$

where the Hamiltonian $H_{q\bar{q}}$ includes all interactions involving the light quarks,

$$H_{q\bar{q}} = -\frac{\nabla^2_{\boldsymbol{r}_{cq}}}{2m_{cq}} - \frac{\nabla^2_{\boldsymbol{r}_{\bar{c}\bar{q}}}}{2m_{cq}} + V^{\rm coul} + V^{\rm conf} + V^{\rm spin}\,, \tag{8}$$

[1] Only the real part of the exponential factor contributes to the amplitude [7].

where m_{cq} is the reduced mass of the cq, which include both the Coulombic and confining components of the gluon-mediated interaction, as well as the spin-dependent terms. For a detailed discussion of the various potentials, we refer to Ref. [12]. The heavy-quark separation $r_{c\bar{c}}$ enters as a parametric dependence. Once the energy shift $\Delta E(r_{c\bar{c}})$ is determined, the heavy-quark wave function $\Psi_{c\bar{c}}$ satisfies

$$[H_{c\bar{c}} + \Delta E(r_{c\bar{c}})]\, \Psi_{c\bar{c}}(\boldsymbol{r}_{c\bar{c}}) = E\, \Psi_{c\bar{c}}(\boldsymbol{r}_{c\bar{c}})\,, \tag{9}$$

where the Hamiltonian $H_{c\bar{c}}$ describes the kinetic energy and interactions of the heavy quarks. The effect of the dynamics of the light quarks, as well as their interactions with the (approximately static) charm quarks, is encoded in the effective potential $\Delta E(r_{c\bar{c}})$.

The light-quark problem in Eq. (7) cannot be solved analytically so we adopt variational methods, decomposing the wavefunction as

$$\Psi_{q\bar{q}}(\boldsymbol{r}_{cq}, \boldsymbol{r}_{\bar{c}\bar{q}}; \boldsymbol{r}_{c\bar{c}}) = a_1(r_{c\bar{c}})\, \psi_D(\boldsymbol{r}_{cq})\psi_D(\boldsymbol{r}_{\bar{c}\bar{q}}) \;+\; a_2(r_{c\bar{c}})\, \psi_M(\boldsymbol{r}_{c\bar{q}})\psi_M(\boldsymbol{r}_{\bar{c}q})\,. \tag{10}$$

The wavefunctions ψ_C with $C = D, M$ are variational test-functions, in the hydrogen-like form

$$\psi_C(\zeta) = \sqrt{\frac{C^3}{\pi}} e^{-C\zeta}\,. \tag{11}$$

The C parameter is chosen to minimize the mean value

$$E_C = \left\langle \psi_C(\zeta) \left| -\frac{\nabla^2}{2m_{cq}} + V_C(\zeta) \right| \psi_C(\zeta) \right\rangle = \frac{C^2}{2m_{cq}} + \lambda_C \alpha_s\, C + \frac{3k}{2C} \tag{12}$$

where

$$V_D(\zeta) = -\frac{1}{3}\alpha_s \frac{1}{\zeta} + k\,\zeta = \lambda_D \alpha_s \frac{1}{\zeta} + k\,\zeta\,, \qquad V_M(\zeta) = -\frac{7}{6}\alpha_s \frac{1}{\zeta} + k\,\zeta = \lambda_M \alpha_s \frac{1}{\zeta} + k\,\zeta\,. \tag{13}$$

The coefficients a_i are determined by solving the variational condition

$$\frac{d}{da_i} \langle \Psi_{q\bar{q}} | H_{q\bar{q}} | \Psi_{q\bar{q}} \rangle = 0\,, \tag{14}$$

which can also be used to compute the Born–Oppenheimer potential $\Delta E(r_{c\bar{c}})$, since Eq. (14) is equivalent to the following generalized eigenvalue problem

$$\begin{pmatrix} H_D - \Delta E(r_{c\bar{c}}) & H_{DM} - S^2_{DM}(r_{c\bar{c}})\, \Delta E(r_{c\bar{c}}) \\ H_{DM} - S^2_{DM}(r_{c\bar{c}})\, \Delta E(r_{c\bar{c}}) & H_M - \Delta E(r_{c\bar{c}}) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{15}$$

where we have introduced the overlap integral

$$S_{DM}(r_{c\bar{c}}) = \int_{\boldsymbol{r}_{cq}} \psi_D(\boldsymbol{r}_{cq})\, \psi_M(\boldsymbol{r}_{cq} - \boldsymbol{r}_{c\bar{c}}) = \int_{\boldsymbol{r}_{\bar{c}\bar{q}}} \psi_D(\boldsymbol{r}_{\bar{c}\bar{q}})\, \psi_M(\boldsymbol{r}_{\bar{c}\bar{q}} - \boldsymbol{r}_{c\bar{c}})\,. \tag{16}$$

The matrix elements appearing in Eq. (15) are discussed in [12]. The BO potential is obtained by imposing the vanishing of the determinant of the matrix, which guarantees the existence of a non-trivial solution (i.e. different from $a_1 = a_2 = 0$). This condition leads to

$$\Delta E^{\pm}(r_{c\bar{c}}) = \frac{1}{2(1 - S^4_{DM})} \Big[H_D + H_M - 2S^2_{DM} H_{DM} \pm \sqrt{\left(H_D + H_M - 2S^2_{DM} H_{DM}\right)^2 - 4(1 - S^4_{DM})(H_D H_M - H^2_{DM})} \Big]\,. \tag{17}$$

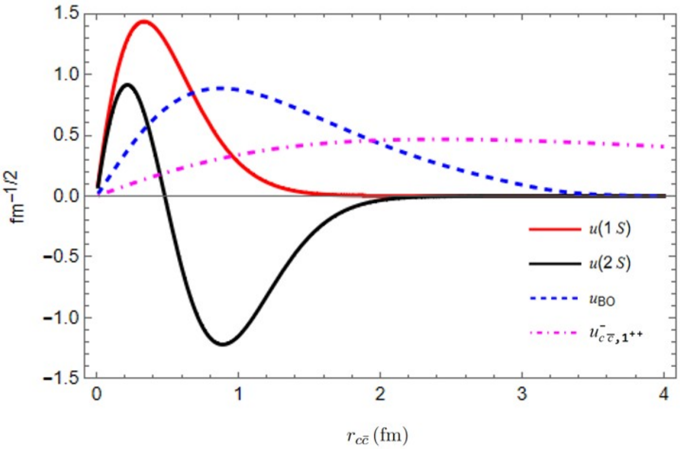

Figure 2: Normalized reduced radial wave functions for the charmonia J/ψ and ψ' and for the $c\bar{c}$ pair: $u_{\rm BO}$ is the wavefunction obtained in [7]. $u^-_{c\bar{c},1^{++}}$ is the solution of (18).

Then we can evaluate also the a_i coefficients. Once the light-quark wave function and the corresponding Born–Oppenheimer potential $\Delta E(r_{c\bar{c}})$ have been determined, the heavy-quark equation (9) can be solved. For the specific case of the $X(3872)$, this equation takes the form

$$\left(-\frac{\nabla^2}{2m_{c\bar{c}}} + \frac{1}{6}\alpha_s \frac{1}{r_{c\bar{c}}} + \Delta E^-_{1^{++}}(r_{c\bar{c}})\right) \Psi_{c\bar{c}}(\boldsymbol{r}_{c\bar{c}}) = E_X \, \Psi_{c\bar{c}}(\boldsymbol{r}_{c\bar{c}}) \,, \tag{18}$$

where $m_{c\bar{c}}$ denotes the reduced mass of the $c\bar{c}$ pair. The potential ΔE^- corresponds to the solution of Eq. (17) with the minus sign in front of the square root, while the subscript 1^{++} indicates the J^{PC} quantum numbers entering the spin-dependent interaction. In Fig. 2 we show the solution of Eq. (18), together with a parallel results obtained in Ref. [7].

4. Conclusion

We are now in a position to compute the ratio in Eq. (5), obtaining

$$\mathcal{R}_{\rm th} = 1.4 \pm 0.3 \,. \tag{19}$$

The uncertainty is estimated by varying the light-quark masses and the string tension of the confining potential by ±10%, which control the spatial extent of the light-quark orbitals and hence the overall size of the tetraquark. As discussed in Ref. [7], the tetraquark size has a strong impact on the ratio $\mathcal{R}$. Our result is in good agreement with the LHCb measurement reported in Eq. (1) [11]. In contrast, molecular interpretations typically predict values below unity [11, 13]. A pure charmonium assignment yields values compatible with experiment [6], but is disfavored by the sizable strong isospin violation observed in the $X \to J/\psi\,\omega$ and $X \to J/\psi\,\rho^0$ decay modes [14].

Beyond this specific observable, the present framework provides a unified description of compact $c\bar{c}q\bar{q}$ states within the Born–Oppenheimer approximation. In particular, the mass spectrum for total angular momentum $J = 0, 1, 2$ can be systematically computed, leading to a pattern consistent with current experimental candidates [12], in line with recent studies by other groups [8].

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