PROCEEDINGS OF SCIENCE

Interdisciplinary Insights into Structural Target Control: Complex Social Networks and Neuronal Dynamics

Klotilda Nikaj[a,*] **Ervis Gega**[b,] **Margarita Ifti**[b]

[a] *Physics Department, University of Shkodra "Luigj Gurakuqi", Sheshi 2 Prilli, Shkoder, Albania*

[b] *Physics Department, University of Tirana, Boulevard Zogu i Zi, Tirane, Albania*

E-mail: klotilda.nikaj@unishk.edu.al, ervis.gega@unitir.edu.al , margarita.ifti@unitir.edu.al

Recent research across complex social networks and neuronal dynamics reveals a multifaceted exploration into emergent behaviors and structural dynamics. In the study of social networks, researchers increasingly utilize advanced network science and computational modeling to analyze dynamic interactions, structural formations, and resilient properties amidst various disruptions. Concurrently, in neuroscience, the study of neuronal dynamics-particularly oscillatory behaviors in neuronal dynamics, analogous to patterns seen in social networks following external stimuli, pose significant theoretical and practical challenges. The self-contained oscillatory patterns and bifurcation phenomena in neuron potential underscore the necessity for detailed nonlinear analysis through numerical integrations and computer simulations across diverse parameter spaces. These simulations explore the concept of structural target control, focusing on strategies to manipulate critical nodes within social networks while considering their inherent structural characteristics. Such simulations elucidate critical insights into system stability, phase space dynamics, and bifurcation diagrams within foundational models like the Fitzhugh- Nagumo (FN) and Hodgkin-Huxley (HH) models, as well as in more advanced interdisciplinary models. These simulations incorporate methodologies from network science and control theory, crucial for identifying minimal sets of influential nodes, or "drivers," capable of steering the dynamics of targeted social network segments. It sets a foundation for future studies that integrate advanced computational techniques with empirical data to address emerging challenges in understanding and managing complex networks.

14th Young Researcher Meeting (14YRM2025)
30 September - 3 October 2025
Gran Sasso Science Institute, L'Aquila

*Speaker

© Copyright owned by the author(s) under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License (CC BY-NC-ND 4.0)
All rights for text and data mining, AI training, and similar technologies for commercial purposes, are reserved.
ISSN 1824-8039 . Published by SISSA Medialab.

PoS(14YRM2025)011

1. Introduction

We are looking to analyze how external stimuli affect the dynamics of a social network [1,2] using a neural network model to simulate neuron responses. In recent years, the intersection of complex social networks and neuronal dynamics has sparked a deeper exploration of the emergent behaviors and structural dynamics that govern these systems. Researchers in both fields increasingly employ sophisticated network science and computational modeling to dissect the intricate interactions, structural formations, and resilience of networks in the face of various disruptions. In the realm of social networks, the analysis of dynamic interactions and structural shifts has become pivotal, particularly when external stimuli, such as advertising payments or promotional campaigns, are introduced. These stimuli can significantly alter node activity and network dynamics, necessitating a robust understanding of how these changes propagate through the network. To analyze how external stimuli affect the dynamics of a social network, a neural network model can be employed to simulate neuron responses.

1.1 Modelling Neurons and Social Networks

In a social network, nodes (individuals) and connections (friends, followers) can be represented as a neural network. In a social network, each node's activity is influenced by both its connections and external stimuli, which can alter network dynamics in response to various inputs. A neural network model [3,4] might incorporate mechanisms to adjust neuron activity based on the stimuli received, along with other mechanisms to simulate behavior within the social network. We extend this model to three specific network structures: directed social networks, where connections have a specified direction; sparse networks, where most nodes have few connections; and fully connected networks, where every node is linked to every other node. These structures are chosen to capture different types of social network configurations and to explore how varying levels of connectivity influence the propagation of external stimuli through the network.

1.2 Change in Flow and Stimuli

External stimuli, such as payments for advertising a product, can alter the “flow” or activity of specific nodes within the network. For instance, if a company compensates an influencer to promote a product, the influencer's followers may exhibit changes in their responses. These changes can be modeled by adjusting the activity levels of neurons representing these individuals in response to the stimulus. From a modeling perspective, such external influences can be represented as time dependent stimuli acting on selected nodes through the term $S_i(t)$ in the dynamical equation, corresponding to advertising campaigns or trending topics in digital platforms. When applied to highly connected or influential nodes, even moderate external input can trigger significant changes due to nonlinear interaction effects. Consequently, the analyses of stimulus driven dynamics can lead to a temporary amplification of engagement or information flow, similar to excitation processes observed in neuronal systems.

PoS(14YRM2025)011

2. Network Dynamics

2.1 Complete Dynamic Activity Equation

To describe the temporal evolution of activity in the network, we consider a continuous-time dynamical system to monitor how the network dynamics evolve by observing variations in connections and node activities. Different neural network models [3,4] can help analyze these dynamics. For example, one might track how an increase in activity in one node influences neighboring nodes and impacts the spread of information or alterations in network structure. The activity of node i at time t is denoted by $x_i(t)$. The dynamics of each node can be influenced by three main contributions: its connections (edges), external stimulus and intrinsic decay. The equation for node dynamics is:

$$\frac{dx_i(t)}{dt} = -a_i x_i(t) + \sum_{j \in N_i} w_{ij} x_j(t) + S_i(t) \quad (1)$$

where a_i is the decay rate for node i, N_i represents the set of nodes connected to node i, w_{ij} is the weight of the connection from node j to node i, $S_i(t)$ is the external stimulus applied to node i at time t. A specific form of this stimulus is given by $S_i(t) = S_0 e^{-\lambda t} u(t)$, where S_0 is the stimulus magnitude, λ is a decay constant controlling how fast the stimulus diminishes over time, and $u(t)$ is a function defining the time-dependent nature of the stimulus. The weights w_{ij} represent the strength of interaction between nodes and may be defined based on network structure or empirical data. This formulation establishes a direct conceptual connection with reduced neuronal models such as the FitzHugh–Nagumo system, where nonlinear interactions between excitation, recovery, and external forcing give rise to excitable dynamics and bifurcation phenomena.

2.2 Connecting Neural and Social Models

Each node in our network represents an individual (or a group of individuals) in the social network, and the activity level of a node represents the level of engagement, influence, or behavior of that individual within the network over time. We'll represent the network as an adjacency matrix, where each element w_{ij} represents the weight of the connection from node *j* to node *i*. The odeint algorithm is a numerical solver for ordinary differential equations (ODEs) that allows us to solve the system of equations governing the dynamics of all nodes in the network simultaneously over time. The function *network dynamics* [2] computes the rate of change of activity for each node, considering the decay, influence from other nodes, and external stimulus. This code will generate a plot showing the activity levels of each node in the network over time. Each curve represents the dynamics of one node, influenced by its connections to other nodes and the external stimulus.

3. Simulation and Analysis

Following the application of stimuli and the observation of changes, analytical methods can be utilized to understand how these stimuli affect the overall dynamics of the network. This may involve tracking changes in node centrality (such as degree centrality or betweenness

PoS(14YRM2025)011

centrality), connection strength (measuring the weight of edges or the number of shared connections), and information distribution (assessed by the spread of influence or information flow across the network). These metrics provide insights into how stimuli impact network structure and behavior. This synthesis of neural network modeling with external stimuli analysis extends to foundational models [3,4] such as the Fitzhugh-Nagumo (FN) and Hodgkin-Huxley (HH) models, as well as more advanced interdisciplinary frameworks.

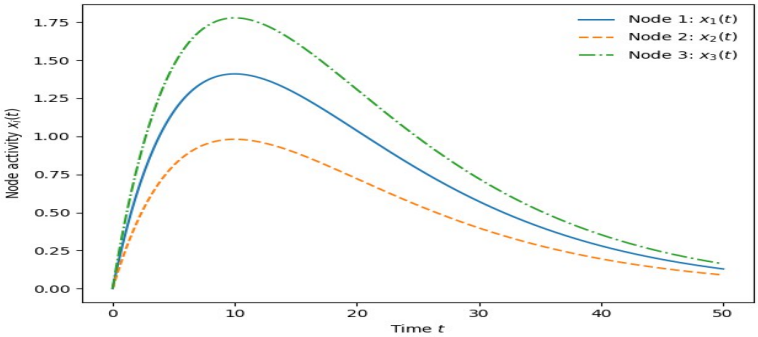

Figure 1 illustrates the temporal evolution of activity over time for three nodes, showing limited propagation and gradual decay due to weak connectivity, where node interactions remain localized and do not lead to sustained amplification.

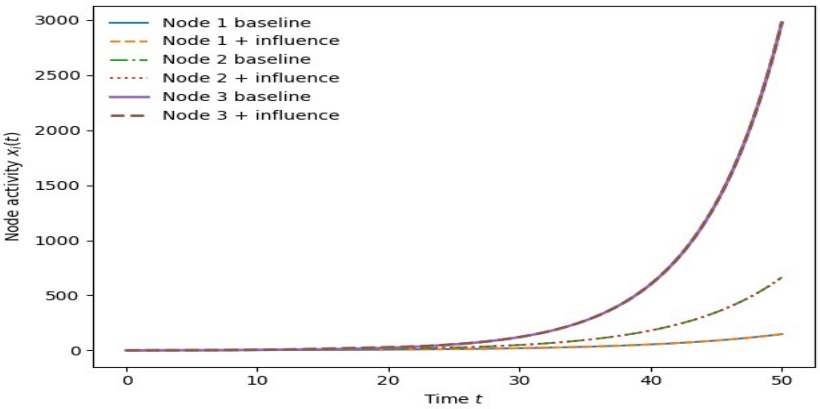

Figure 2. Temporal evolution of node activity in a directed network with asymmetric connections, illustrating non-reciprocal influence and amplification effects induced by external stimuli.

The directed and asymmetric connections (see Figure 1) create feedback loops. In the absence of external stimulus node activity follows a transient growth, followed by relaxation toward equilibrium. For example, the influence from Node 1 to Node 2, from Node 2 to Node 3, and from Node 3 back to Node 1 suggests that actions and behaviors in the network can cycle and amplify over time. This could represent a scenario where social behaviors or information circulate within a closed group, reinforcing the group's dynamics. The sparse Network results in weak connectivity: Limited interaction and influence between nodes result in isolated or minimal social dynamics. In a real-world scenario, this might correspond to a network where individuals are loosely connected, with little information or behavior spreading through the network. Influence is concentrated and does not propagate far leading to isolated pockets of activity.

PoS(14YRM2025)011

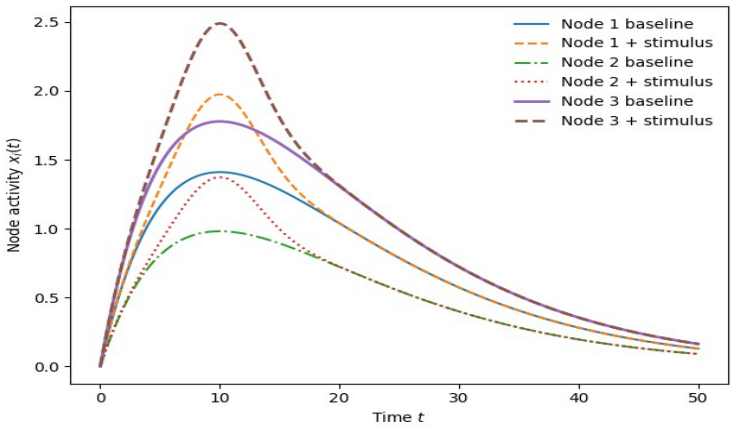

Figure 3. Temporal evolution of node activity in a directed asymmetric network under an external stimulus effect.

In figure 3 is shown a directed networks with assimetric connections, where we can notice that asymmetric connectivity promotes faster growth along dominant influence pathways. External stimulus produces a temporary amplification before the networks returns toward equilibrium.

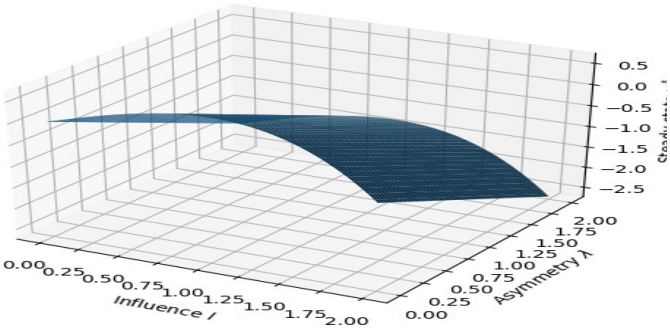

Figure 4: Three-dimensional bifurcation diagram of the system dynamics as a function of the external stimulus, showing changes in stability and the onset of oscillatory behavior associated with Hopf bifurcations.

As seen in Figure 4, the system exhibits excitable behaviour in social response dynamics. The response surface represents the steady-state activity x* as a function of two competing factors:

- the external influence strength I
- the structural or regulatory parameter λ

The shape of the surface reflects the hallmark behavior of excitable systems. For small values of III, the system remains in a low-activity state regardless of λ. Here the external input is insufficient to trigger a large response acting as a mirror for the resting state in FitzHugh–Nagumo dynamics. As influence strength increases, a critical transition region appears, and even a modest increase in influence suddenly triggers large engagement. influence does not merely perturb the system — it reorganizes its steady state. The surface reveals that: amplification is not purely driven by input strength, but emerges from the interplay between external forcing and internal recovery dynamics.

PoS(14YRM2025)011

4. Results and discussions

The phenomenon of neuronal oscillation exhibits interesting behaviour, the dynamics of which consists of fixed points, oscillations and Hopf bifurcations depending on the control parameter, which in our case is the external stimulus that we give to certain nodes. The presented simple mathematical models give us acceptable information on phenomenon previously experimented in real cases. The results indicate that network response is governed not only by the magnitude of external stimulus, but also by network structure. Directed assymetry enhances the system toward higher activity states.

From the simplified case FitzHugh–Nagumo we observe that the system exhibits excitable behavior, where small inputs , under suitable structural conditions can trigger large scale activity.

5. Conclusions

The integration of neuronal models with social network analysis offers a promising avenue for understanding and predicting the behaviour of complex systems. By simulating how individuals respond to stimuli within various network structures, we can gain valuable insights into the underlying dynamics of social systems and design more effective strategies for influencing or optimizing these networks. The approach holds significant potential for a wide range of applications, from targeted marketing to social policy, and provides a rich framework for future interdisciplinary research.

References

[1] S.H. Strogatz, *Exploring complex networks*, Nature **410** (2001) 268–276.

[2] A.-L. Barabási and R. Albert, *Complex networks*, Science **292** (2003) 2323–2324.

[3] R. FitzHugh, *Impulses and physiological states in theoretical models of nerve membrane*, Biophysical Journal **1** (1961) 445–466.

[4] A.L. Hodgkin and A.F. Huxley, *A quantitative description of membrane current and its application to conduction and excitation in nerve*, Journal of Physiology **117** (1952) 500–544.

[5] L.F. Shampine and M.W. Reichelt, *The MATLAB ODE suite*, SIAM Journal on Scientific Computing **18** (1997) 1–22.

[6] E. Hopf, *A generalization of the Poincaré–Lyapunov criterion for periodic solutions of differential equations*, Mathematische Annalen **125** (1948) 1–20.

PoS(14YRM2025)011