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Resurgent Lambert series from Feynman and beyond

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Lambert series of the form $\sum_{n>0} a(n)q^n/(1-q^n)$ are ubiquitous in mathematical physics. In particular, 2-loop sunrise and 3-loop banana Feynman diagrams yield Lambert series with $a(n)$ of the form $\chi(n)/n^s$ where $\chi(n)$ is a Dirichlet character. Resurgence concerns the singular limit as $|q|$ approaches 1. In the Feynman cases we can control this limit, obtaining rapidly convergent expressions, since the Lambert series are iterated integrals of holomorphic Eisenstein series twisted by a character. We generalize this result, to include modular resurgent structures found in topological-string observables.

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1. Introduction

We begin with an identity for $\zeta(3)$ from Ramanujan's notebooks. It comes from a more general formula that lay there undiscovered and unproved for 50 years [5, 18]. In Section 2, we show how this relates the appearance of $\zeta(3)$ in the behaviour of a 3-loop Feynman integral [7] at small and large momenta. In Section 3, we explain how this may be regarded as an example of Cheshire cat resurgence [13], with exponentially suppressed terms surviving when a perturbative series terminates. We also show that a 2-loop integral benefits from this simplicity, thanks to the inclusion of a quadratic character in a Lambert series. In Section 4, we outline a more general result from [10], where we considered a pair of characters and found the non-perturbative terms that resolve ambiguities of resummation. Finally, in Section 5, we apply our analysis to computations of spectral traces in topological string theory [15–17, 20]. We find that compactification on a Calabi-Yau manifold with $\mathbb{P}^{m,n}$ geometry results in appearance of Lambert series twisted by characters with conductor $N = m + n + 1$. At $N = 5$, we encounter both odd and even characters and hence an admixture of a more difficult case, requiring Borel resummation, with a simpler case, exhibiting Cheshire cat resurgence.

2. Ramanujan's transformation of the 3-loop banana

Ramanujan's notebooks [5] give an efficient way of evaluating $\zeta(3)$ from the Lambert series in

$$\zeta(3) = \frac{7\pi^3}{180} - 2\sum_{n>0}\frac{1}{n^3}\frac{q^n}{1-q^n}, \quad q = \exp(-2\pi) < \frac{1}{353}, \tag{1}$$

which is the first of his identities

$$\zeta(4k-1) = \frac{1}{\pi}\sum_{n=0}^{2k}(-1)^{n+1}\zeta(2n)\,\zeta(4k-2n) - 2\sum_{n>0}\frac{n^{-4k+1}}{\exp(2\pi n)-1}\,. \tag{2}$$

Similarly $\zeta(5)$ is rapidly evaluated by setting $k = 1$ in the identity [3]

$$\begin{aligned} k\,\zeta(4k+1) = {} & \frac{1}{\pi}\sum_{n=0}^{2k+1}(-1)^n\,(n-\tfrac{1}{2})\,\zeta(2n)\,\zeta(4k+2-2n) \\ & - 2\sum_{n>0}\frac{n^{-4k-1}}{\exp(2\pi n)-1}\left(k + \frac{\pi n}{1-\exp(-2\pi n)}\right). \end{aligned} \tag{3}$$

We can improve [9] upon (1), by evaluating Lambert series with $q \le \exp(-12\pi/\sqrt{15}) < \frac{1}{16879}$ in

$$\zeta(3) = \frac{16351\pi^3}{330^2\sqrt{15}} + \frac{2}{121}W_{12} + \frac{1368}{605}W_{15} - \frac{6}{605}W_{20} - \frac{2924}{605}W_{30} + \frac{342}{605}W_{60}\,, \tag{4}$$

$$W_k = \sum_{n>0}\frac{1}{n^3}\frac{1}{\exp(\pi k n/\sqrt{15})-1}. \tag{5}$$

Ramanujan's value of $\zeta(3)$ came from a family of quasi-modular transformations of

$$T_s(\tau) = \frac{1}{2}\zeta(s) + \sum_{n>0}\frac{1}{n^s}\frac{q^n}{1-q^n}, \quad q = \exp(2\pi i\tau) = \exp(-2\pi y) \tag{6}$$

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with $\Im\tau = \Re y > 0$. This converges well at large y. To access $|q| \to 1^-$, as $y \to 0^+$, we use a Fricke involution $y \to 1/y$, when s is odd. For example, Ramanujan obtained [5, 12]

$$yT_3(\mathrm{i}/y) = \frac{\pi^3}{180}\left(y^2 + 5 + \frac{1}{y^2}\right) - \frac{T_3(\mathrm{i}y)}{y} \tag{7}$$

$$y^2T_5(\mathrm{i}/y) = \frac{\pi^5}{3780}\left(2y^3 + 7y - \frac{7}{y} - \frac{2}{y^3}\right) + \frac{T_5(\mathrm{i}y)}{y^2} \tag{8}$$

$$y^3T_7(\mathrm{i}/y) = \frac{\pi^7}{56700}\left(3y^4 + 10y^2 - 7 + \frac{10}{y^2} + \frac{3}{y^4}\right) - \frac{T_7(\mathrm{i}y)}{y^3} \tag{9}$$

which originate from Eichler integration of holomorphic Eisenstein series

$$\sum_{n>0}\frac{n^s q^n}{1-q^n} = \left(q\frac{\mathrm{d}}{\mathrm{d}q}\right)^s \sum_{n>0}\frac{n^{-s}q^n}{1-q^n} \tag{10}$$

with even modular weights $s + 1 \geq 4$. Iterated integrals of Eisenstein series and other modular forms have been intensively studied in quantum field theory [2, 11] and in string theory [14].

The Schwinger parametrization of the equal-mass 3-loop banana integral in two spacetime dimensions gives [7]

$$J(t) = \int_0^\infty\int_0^\infty\int_0^\infty \left(\frac{1}{(1+a+b+c)(1+1/a+1/b+1/c)-t}\right)\frac{\mathrm{d}a}{a}\frac{\mathrm{d}b}{b}\frac{\mathrm{d}c}{c} \tag{11}$$

and for real $t < 16$ it is given by the Bessel moment [4]

$$J(t) = 8\int_0^\infty I_0(\sqrt{t}x)K_0^4(x)x\,\mathrm{d}x. \tag{12}$$

Broadhurst and Laporta discovered the on-shell evaluation [4]

$$J(1) = \frac{\pi^3}{2}\left(1-\frac{1}{\sqrt{5}}\right)\left(1+2\sum_{n>0}\exp(-\sqrt{15}\pi n^2)\right)^4 = \frac{\Gamma\left(\frac{1}{15}\right)\Gamma\left(\frac{2}{15}\right)\Gamma\left(\frac{4}{15}\right)\Gamma\left(\frac{8}{15}\right)}{30\sqrt{5}}, \tag{13}$$

which was later proved by work in [7], completed in [21].

To develop the Taylor series $J(t) = \sum_{n\geq 0} C_n t^n$, one may use the recurrence relation [4, 8]

$$C_n = \left(1-\frac{1}{2n}\right)\left(5-\frac{5}{n}+\frac{2}{n^2}\right)C_{n-1} - 4\left(1-\frac{1}{n}\right)^3 C_{n-2}. \tag{14}$$

for $n > 1$, with initial conditions $C_0 = 7\zeta(3)$ and $C_1 = C_0 - 6$ that ensure that $J(4)$ is finite. This recurrence relation determines the inhomogeneous third-order Picard-Fuchs equation

$$t^2(t-4)(t-16)J''' + 6t(t^2-15t+32)J'' + (7t^2-68t+64)J' + (t-4)J + 24 = 0 \tag{15}$$

with singularities for $t \in \{0, 4, 16, \infty\}$.

A modular parametrization of the homogeneous equation was given by Helena Verrill [22] in 1996, using the Dedekind eta function

$$\eta(\tau) = q^{1/24}\prod_{n=1}^\infty(1-q^n) = \sum_{n=-\infty}^\infty (-1)^n q^{(6n+1)^2/24} = \frac{\eta(-1/\tau)}{\sqrt{-\mathrm{i}\tau}} \tag{16}$$

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where $q = \exp(2\pi i\tau)$. Denoting $\eta_n := \eta(n\tau)$, let

$$t = -64\left(\frac{\eta_2\eta_6}{\eta_1\eta_3}\right)^6, \quad \psi = \frac{\eta_1^2\eta_3^2}{\eta_2\eta_6}, \quad h = \frac{\eta_2^{16}}{\eta_1^8} - 9\frac{\eta_6^{16}}{\eta_3^8} \tag{17}$$

where ψ^2 is the square of a elliptic integral, obtained by Geoff Joyce [19] in 1973. as a solution to the homogeneous equation, and h is a weight-4 cuspform of level 6 [1]. Then we obtain the inhomogeneous equation

$$\left(q\frac{\mathrm{d}}{\mathrm{d}q}\right)^3 \frac{J(t)}{\psi^2} = 24h = 24\sum_{n=1}^{\infty} \frac{n^3(q^n - 8q^{3n} + q^{5n})}{1 - q^{6n}} \tag{18}$$

which is easily integrated to give Lambert series in

$$\frac{J(t)}{\psi^2} = H(\tau) = 24T_3(\tau) - 3T_3(2\tau) - 8T_3(3\tau) + T_3(6\tau). \tag{19}$$

The Lambert series converge well for $|t| \le 8$ and at $t = 0$ give the correct value of

$$J(0) = \tfrac{1}{2}(24 - 3 - 8 + 1)\zeta(3) = 7\zeta(3). \tag{20}$$

The Fricke involution $\tau \to -1/(6\tau)$ gives $t \to 64/t$ and Ramanujan delivers

$$48\tau^2 H\left(\frac{-1}{6\tau}\right) = 4(2\pi i\tau)^3 + 16(3T_3(\tau) - 6T_3(2\tau) - T_3(3\tau) + 2T_3(6\tau)) \tag{21}$$

for fast evaluation of $J(t)$ with $|t| \ge 8$. Then, at large $|t|$, we obtain

$$-tJ(t) = 4\log^3(-t) + 16\zeta(3) + O(\log^3(-t)/t) \tag{22}$$

whose asymptotic constant $16\zeta(3)$ is determined by $J(0) = 7\zeta(3)$. Thus Ramanujan's notebooks, from more than 100 years ago, provide us with a connection between expansions at large and small momenta. In Section 5 we use a Fricke involution to relate expansions at strong and weak couplings for topological string observables.

3. Cheshire cat resurgence

> *"Well! I've often seen a cat without a grin," thought Alice; "but a grin without a cat! It's the most curious thing I ever saw in my life!"* (Charles Lutwidge Dodgson, alias Lewis Carroll, in *Alice's Adventures in Wonderland*, 1865.)

Ramanujan's very useful formula

$$T_{2k+1}\left(\frac{-1}{\tau}\right) = \frac{\mathrm{i}}{\pi}\sum_{j=0}^{k+1} \frac{\zeta(2j)\zeta(2k+2-2j)}{\tau^{2j-1}} + \frac{T_{2k+1}(\tau)}{\tau^{2k}} \tag{23}$$

has a terminating Laurent series and an exponentially suppressed tail, as $\tau \to \mathrm{i}\infty$. By contrast, Lambert series $\sum_{n>0} n^{-s}q^n/(1-q^n)$ with even s give divergent series, requiring directional Borel

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resummation whose ambiguities are eventually resolved by careful treatment of the exponentially suppressed tail.

In [13] the quasi-modularity of Ramanujan's formula for odd s was described as an example of "Cheshire cat resurgence", since exponentially suppressed terms remain after the necessity of resummation of the perturbative terms has disappeared. Amusingly, we know of a Cheshire cat with even s. The 2-loop sunrise diagram produces Lambert series with $s = 2$. So how can it avoid a divergent asymptotic series at large momenta?

For the sunrise integral $I(w^2) = 4\int_0^\infty I_0(wx)K_0^3(x)x\,dx$, the modular parametrization [1, 6]

$$w = 3\frac{\eta_2^2\eta_3^4}{\eta_1^4\eta_6^2}, \quad f = \frac{\eta_1^6\eta_6}{\eta_2^3\eta_3^2}, \quad g = \frac{\eta_3^9}{\eta_1^3} + \frac{\eta_6^9}{\eta_2^3} \tag{24}$$

neatly reduces the problem to solving

$$-\left(q\frac{\mathrm{d}}{\mathrm{d}q}\right)^2\frac{I(w^2)}{f} = 6g = 6\sum_{n=1}^\infty \frac{n^2(q^n - q^{5n})}{1-q^{6n}} \tag{25}$$

and the integration produces, inter alia, a Lambert series [6]

$$\mathcal{L}_2(\chi_{6,5};q) = \sum_{n=1}^\infty \frac{\chi_{6,5}(n)}{n^2}\frac{q^n}{1-q^n} \tag{26}$$

with an odd character $\chi_{6,5}(n) = \pm 1$ for $n = \pm 1 \bmod 6$ and $\chi_{6,5}(n) = 0$ otherwise. Then quasi-modularity [1] allows one to expand about any of the singular points $w \in \{0, 1, 3, \infty\}$ that locate cusps of modular forms on the congruence subgroup $\Gamma_0(6)$.

3.1 Two types of divisor sums

The Lambert series

$$\begin{aligned}\mathcal{L}_s(\chi_r;\tau) &= \sum_{n>0}\frac{\chi_r(n)}{n^s}\frac{q^n}{1-q^n}\\ &= \sum_{n>0}\frac{\sigma'_{s,\chi_r}(n)\,q^n}{n^s}, \quad \sigma'_{s,\chi_r}(n) = \sum_{d|n}\chi_r(n/d)d^s,\end{aligned} \tag{27}$$

has a Fourier expansion whose n-th coefficient involves sums of powers of divisors $d|n$ weighted by the character $\chi_r(n/d)$. We shall also need the dual series

$$\widetilde{\mathcal{L}}_s(\chi_r;\tau) = \sum_{n>0}\frac{\sigma_{s,\chi_r}(n)\,q^n}{n^s}, \quad \sigma_{s,\chi_r}(n) = \sum_{d|n}\chi_r(d)d^s. \tag{28}$$

If χ_D is the primitive quadratic character with fundamental discriminant D, then we have quasi-modularity, for odd s when $D > 1$, and for even s, when $D < 0$. With conductor $N = |D|$, the modularity gap is given by evaluations of Bernoulli polynomials, with

$$\begin{aligned}&\sqrt{D}\left(2\mathcal{L}_s(\chi_D;\tau) + \sum_{n>0}\frac{\chi_D(n)}{n^s}\right) + 2(-\tau)^{s-1}\widetilde{\mathcal{L}}_s\left(\chi_D;\frac{-1}{N\tau}\right) =\\ &\frac{(2\pi\mathrm{i})^s}{(s+1)!}\sum_{j=0}^{\lfloor s/2\rfloor}\tau^{2j-1}\binom{s+1}{2j}B_{2j}(0)\sum_{m=1}^{N-1}\chi_D(m)B_{s+1-2j}(m/N).\end{aligned} \tag{29}$$

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4. The general case: resolving an ambiguity in resummation

In [10] we generalized by including pairs of characters and indices in

$$\Xi_{s_1,s_2}(\chi_{r_1},\chi_{r_2};\tau)=\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\frac{\chi_{r_1}(n_1)}{n_1^{s_1}}\frac{\chi_{r_2}(n_2)}{n_2^{s_2}}q^{n_1n_2}. \tag{30}$$

With $\tau=\mathrm{i}y$ and $y\to 0^+$ we found that resummed perturbative terms require, in general, an exponentially suppressed tail that is a sum of transformed terms:

$$\sum_{n=0}^{\infty}\frac{(s_1)_n(s_2)_n}{n!}\frac{(r_1r_2\tau)^{s_1+s_2+n-1}}{(-2\pi\mathrm{i})^n}\,\Xi_{s_1+n,s_2+n}\left(\bar\chi_{r_2},\bar\chi_{r_1};\frac{-1}{r_1r_2\tau}\right) \tag{31}$$

with only one term surviving in the single character cases

$$\mathcal{L}_s(\chi_r;\tau)=\Xi_{s,0}(\chi_r,\chi_{1,1};\tau)=\Xi_{0,s}(\chi_{1,1},\chi_r;\tau) \tag{32}$$

$$\widetilde{\mathcal{L}}_s(\chi_r;\tau)=\Xi_{0,s}(\chi_r,\chi_{1,1};\tau)=\Xi_{s,0}(\chi_{1,1},\chi_r;\tau) \tag{33}$$

relevant to work by Veronica Fantini and Claudia Rella on quantum modularity of resurgent topological strings [15–17, 20].

5. Topological strings

These authors studied a fermionic spectral trace, in topological string theory compactified on a Calabi-Yau manifold with $\mathbb{P}^2$ geometry, obtaining q-Pochhammer symbols in

$$\mathrm{Tr}(\rho_{\mathbb{P}^2})=\frac{1}{3\sqrt{\tau}}\exp\left(-\frac{\pi\mathrm{i}}{12}\tau+\frac{\pi\mathrm{i}}{36}\tau^{-1}+\frac{\pi\mathrm{i}}{4}\right)\frac{(q^2;q^3)_\infty^2}{(q;q^3)_\infty}\,\frac{(\omega_3;\tilde q)_\infty}{(\omega_3^{-1};\tilde q)_\infty^2}, \tag{34}$$

$$(xq^\alpha;q)_\infty:=\prod_{n=0}^{\infty}\left(1-xq^{\alpha+n}\right),\quad q:=\exp(2\pi\mathrm{i}\tau),\quad \tilde q:=\exp(-2\pi\mathrm{i}/(3\tau)) \tag{35}$$

where $\omega_3=\exp(2\pi\mathrm{i}/3)$ is a primitive cube root of unity.

Taking a logarithm, we were able to simplify this result, obtaining

$$\log\left[\mathrm{Tr}(\rho_{\mathbb{P}^2})\right]=-\frac{1}{2}\log(3^{\frac{5}{2}}\tau)-\frac{\pi\mathrm{i}}{4}+\frac{3}{2}\left(\widetilde{\mathcal{L}}_1(\chi_{3,2};\tau)-\sqrt{-3}\mathcal{L}_1(\chi_{3,2};-\tfrac{1}{3\tau})\right) \tag{36}$$

with the odd character $\chi_{3,2}(n)=\pm1$ for $n=\pm1$ mod 3 and $\chi_{3,2}(n)=0$ otherwise.

At large or small coupling $y=-\mathrm{i}\tau$ this requires resummation of an infinite number of perturbative terms, with an exponentially suppressed tail in

$$\begin{pmatrix}\mathcal{L}_1(\chi_{3,2};\tau)\\ \widetilde{\mathcal{L}}_1(\chi_{3,2};\tau)\end{pmatrix}=\begin{pmatrix}\mathcal{S}_\mp\left[\mathcal{L}_1^{\mathrm{Pert}}\right](\chi_{3,2};\tau)\\ \mathcal{S}_\mp\left[\widetilde{\mathcal{L}}_1^{\mathrm{Pert}}\right](\chi_{3,2};\tau)\end{pmatrix}\pm\mathrm{i}\begin{pmatrix}0&\frac{1}{\sqrt3}\\ \sqrt3&0\end{pmatrix}\begin{pmatrix}\mathcal{L}_1(\chi_{3,2};-\frac{1}{3\tau})\\ \widetilde{\mathcal{L}}_1(\chi_{3,2};-\frac{1}{3\tau})\end{pmatrix} \tag{37}$$

resolving the ambiguity of directional Borel resummation, indicated by $\mathcal{S}_\mp$.

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5.1 Spectral trace of local $\mathbb{P}^{m,n}$

Here we need to consider characters with conductor $N = m + n + 1$. With $(m, n) = (2, 1)$, we encounter the odd character $\chi_{4,3}(n) = \pm 1$ for $n = \pm 1 \bmod 4$ and $\chi_{4,3}(n) = 0$ otherwise. The logarithm of the spectral trace involves the combination

$$\widetilde{\mathcal{L}}_1(\chi_{4,3};\tau) - \sqrt{-4}\mathcal{L}_1(\chi_{4,3};-\tfrac{1}{4\tau}) \tag{38}$$

and hence directional Borel resummation is required at both large and small coupling.

With $(m, n) = (3, 1)$, or $(m, n) = (2, 2)$, there are three non-principal characters to consider:

$n \bmod 5$	1	2	3	4
$\chi_{5,2}(n)$	1	i	−i	−1
$\chi_{5,3}(n)$	1	−i	i	−1
$\chi_{5,4}(n)$	1	−1	−1	1

The odd quartic characters give contributions that require Borel resummation, while the even quadratic character gives a contribution

$$\widetilde{\mathcal{L}}_1(\chi_{5,4};\tau) - \sqrt{5}\mathcal{L}_1(\chi_{5,4},-\tfrac{1}{5\tau}) \tag{39}$$

with perturbative expansions that terminate at both large and small coupling. Yet a non-perturbative tail persists, like the grin of Lewis Carroll's Cheshire cat at sunrise.

References

[1] Kevin Acres and David Broadhurst, "Eta quotients and Rademacher sums", in *Elliptic Integrals, Elliptic Functions and Modular Forms in Quantum Field Theory* (ed. Blümlein et al.) Springer Nature (2019) pp 1–27, [arXiv:1810.07478].

[2] Luise Adams and Stefan Weinzierl, "Feynman integrals and iterated integrals of modular forms", Commun. Number Theory Phys. **12** (2018) 193–251, [arXiv:1704.08895].

[3] David H. Bailey and David Broadhurst, "A seventeenth-order polylogarithm ladder", [arXiv:math/9906134].

[4] David H. Bailey, Jonathan M. Borwein, David Broadhurst and M.L. Glasser, "Elliptic integral evaluations of Bessel moments", J. Phys. A: Math. Theor. **41** (2008) 205203, [arXiv:0801.0891].

[5] Bruce C. Berndt, "Modular transformations and generalizations of several formulæ of Ramanujan", Rocky Mountain J. Math. **7** (1977) 147–189.

[6] Spencer Bloch and Pierre Vanhove, "The elliptic dilogarithm for the sunset graph", J. Number Theor. **148** (2015) 328–364, [arXiv:1309.5865].

[7] Spencer Bloch, Matt Kerr and Pierre Vanhove, "A Feynman integral via higher normal functions", Compositio Math. **151** (2015) 2329–2375, [arXiv:1406.2664].

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[8] Jonathan Borwein and Bruno Salvy, "A proof of a recurrence for Bessel moments", Exp. Math. **1** (2008) 223–230, [arXiv:0706.1409].

[9] David Broadhurst, "Five families of rapidly convergent evaluations of zeta values", [arXiv:2401.08997].

[10] David Broadhurst and Daniele Dorigoni, "Resurgent Lambert series with characters", [arXiv:2507.21352].

[11] Johannes Broedel, Claude Duhr, Falko Dulat, Brenda Penante and Lorenzo Tancredi, "Elliptic symbol calculus: from elliptic polylogarithms to iterated integrals of Eisenstein series", JHEP **1808** (2018) 014 [arXiv:1803.10256].

[12] Sarth Chavan, "An elementary proof of Ramanujan's identity for odd zeta values", J. Classical Analysis **19** (2022) 139–147, [arXiv:2104.13223].

[13] Daniele Dorigoni and Axel Kleinschmidt, "Resurgent expansion of Lambert series and iterated Eisenstein integrals", Commun. Number Theory Phys. **15** (2021) 1–57, [arXiv:2001.11035].

[14] Daniele Dorigoni, Mehregan Doroudiani, Joshua Drewitt, Martijn Hidding, Axel Kleinschmidt, Nils Matthes, Oliver Schlotterer and Bram Verbeek, "Modular graph forms from equivariant iterated Eisenstein integrals", JHEP **12** (2022) 162, [arXiv:2209.06772].

[15] Veronica Fantini and Claudia Rella, "Strong-weak symmetry and quantum modularity of resurgent topological strings on local $\mathbb{P}^2$", Commun. Number Theory Phys. **19** (2025) 1–73, [arXiv:2404.10695].

[16] Veronica Fantini and Claudia Rella, "Modular resurgent structures," [arXiv:2404.11550].

[17] Veronica Fantini and Claudia Rella, "Modular resurgence, q-Pochhammer symbols, and quantum operators from mirror curves", Lett. Math. Phys. **116** (2026) 39, [arXiv:2506.08265].

[18] Emil Grosswald, " Comments on some formulæ of Ramanujan", Acta Arith. **21** (1972) 25–34.

[19] G.S. Joyce, "On the simple cubic lattice Green function", Phil. Trans. Roy. Soc. Math. Phys. Sci. **273** (1973) 583–610.

[20] Claudia Rella, "Resurgence, Stokes constants, and arithmetic functions in topological string theory", Commun. Number Theory Phys. **17** (2023) 709–820, [arXiv:2212.10606].

[21] Detchat Samart, "Feynman integrals and critical modular L-values", Commun. Number Theory Phys. **10** (2016) 133–156, [arXiv:1511.07947].

[22] H. A. Verrill, "Root lattices and pencils of varieties", J. Math. Kyoto Univ. **6** (1996) 423–446.

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