PROCEEDINGS OF SCIENCE

ggxy: Fast and flexible NLO QCD corrections to gluon-initiated processes

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We present the C++ library ggxy, which can be used for the fast and flexible calculation of partonic and hadronic cross sections to gluon-initiated top-mediated processes such as $gg \to HH$, as well as the newly derived processes $gg \to ZH$ and $gg \to ZZ$, at NLO QCD including full top-quark mass dependence. The two-loop virtual amplitudes are implemented using analytical approximations in different kinematic regions, while all other parts of the calculation are exact. This implementation allows to freely modify all input parameters, such as the top-quark mass and its renormalization scheme and the masses of the external Z/Higgs bosons. In addition, ggxy has been interfaced to Powheg, which allows the matching to parton showers.

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1. Introduction

The continuos improvements on the experimental and the greatly reduced statistical uncertainties expected from the High-Luminosity LHC (HL-LHC) requires also improvements on the theoretical side to reduce uncertainties to exploit the full potential of new measurements. This on the one hand requires the calculation of difficult higher-order corrections and on the other hand to make the numerical evaluation of them efficient and stable.

In these proceedings we discuss the calculation of the two-loop QCD corrections with full top-quark mass effects of $gg \to ZH$ and $gg \to Z^\star Z^\star$. The results of $gg \to ZH$ have been implemented to the public C++ library `ggxy` [1, 2] and provide a fast and stable numerical evaluation of the amplitudes and hadronic cross sections. This is achieved by the computation of the two-loop amplitudes in the high-energy and the forward limits that combined cover the whole phase space [3]. In addition, we use these results to implement the $gg \to \ell^-\ell^+ H$ and $gg \to \nu_\ell \bar{\nu}_\ell H$ process at NLO QCD preserving spin correlations and off-shell effects of the Z decay in `POWHEG` [4], which can be used for the combination with parton showers from e.g. `Pythia` [5] or `Herwig` [6].

While our results implement in `ggxy` represents the first public available implementation of $gg \to ZH$ at NLO QCD with full top-quark mass dependence, the two-loop corrections have also been calculated using different methods. In particular, the loop-induced LO order contribution has been calculated in Refs. [7–10]. The NLO QCD corrections have been first calculated in the large-mass expansion [11–15] followed by calculations in the high-energy expansion [14, 16], the small-mass expansion [17] and the p_T expansion [18, 19]. In contrast to these expansion methods, the two-loop amplitudes have also been computed in Refs. [20–23] with numerical methods. The two-loop corrections to $gg \to ZZ$ induced by a light-quark loop were computed in Refs. [24, 25].

These proceedings are organized as follows: In Section 2 we discuss the computation of the two-loop amplitudes of both processes using the high-energy and forward expansions. Numerical results of the two-loop amplitudes are presented in Section 3. In Section 4 we describe the implementation of the $gg \to ZH$ process in `ggxy`.

2. Technical details

The main bottleneck in the calculation of NLO QCD corrections with full top-quark mass dependence for $gg \to ZH$ and $gg \to Z^\star Z^\star$ is the computation of the two-loop amplitudes. This is circumvented by the computation of those amplitudes in the high-energy and the forward limits [3] that not only cover together the whole phase space but also provide an efficient and numerically stable way to compute the two-loop amplitudes as already demonstrated for the simpler $gg \to HH$ process [1]. The starting point of both expansions is the same that the amplitudes are expanded in the limit where the external masses of final-state particles, q_3^2 and q_4^2, are smaller than all other scales involved in the process. This expansion corresponds to a Taylor expansion so that the amplitudes can afterwards be written as a sum of master integrals ($M_i(s,t,m_t^2,\epsilon)$) that do not depend on the external masses. The master integrals are next expanded in the two different limits, where in the forward expansion we consider $|t| \ll s, m_t^2$ and in the high-energy expansion we have $m_t^2 \ll s, |t|$. While the former expansion is again a Taylor expansion, in the high-energy limit we have to consider an asymptotic expansion. The results of the forward limit can alternatively be

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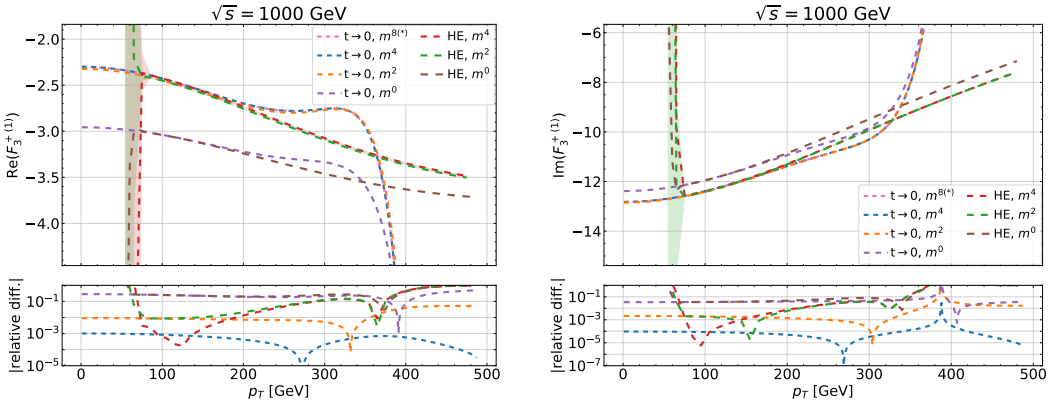

Figure 1: Real and imaginary parts of $F_3^{+(1)}$ for $gg \to ZH$ as a function of p_T for $\sqrt{s} = 1000$ GeV. High-energy and $t \to 0$ expansions are shown including mass corrections up to $m_{Z,H}^{\{0,2,4\}}$. Also shown are higher mass corrections up to $m_{Z,H}^{8}$ for the $t \to 0$ expansion with fewer expansion terms in t according to $t^{n_t}(m_Z^2)^{n_3}(m_H^2)^{n_4}$ with $n_t + n_3 + n_4 \leq 4$ denoted by $\star$. Lower panels display the relative difference with respect to the best approximation of the $t \to 0$ expansion. Figure taken from Ref. [3].

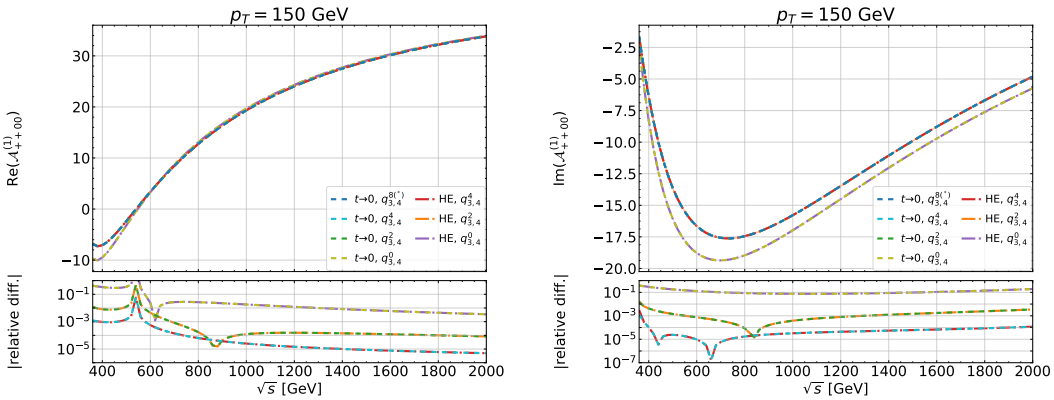

Figure 2: Same as in Figure 1 for the helicity amplitude $\mathcal{A}_{++++}^{(1)}$ for $gg \to Z^\star Z^\star$ as a function of $\sqrt{s}$ for $p_T = 150$ GeV. Figure taken from Ref. [3].

obtained by performing an expansion in the small variable δq that is introduced in the amplitude with $q_3 = -q_1 + \delta q$, where q_3 is the momentum of one of the two final-state particles and q_1 of one of the two gluons. This method leads to a simultaneous expansion in the small variables t, q_3^2 and q_4^2 so that a deeper expansion in the external masses is possible but less terms in the t expansion can be achieved compared to the first method. In addition, we construct a Padé approximation for the amplitudes obtained in the high-energy expansion following Ref. [16] to increase the region of convergence.

3. Numerical results for two-loop amplitudes

In order to demonstrate the general convergence pattern of both expansion methods, we show in Figure 1 the real and imaginary parts of the form factor $F_3^{+(1)}$ for the $gg \to ZH$ process as function of p_T for $\sqrt{s} = 1000$ GeV. Results are shown for the high-energy and forward ($t \to 0$) expansions

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$\sqrt{s}$		ggxy	Ref. [35]
13 TeV	σ^{LO} [fb]	$63.97(2)^{+26.7\%}_{-20.0\%}$	$64.0^{+27\%}_{-20\%}$
	σ^{NLO} [fb]	$118.40(8)^{+16.5\%}_{-14.0\%}$	$118.4^{+17\%}_{-14\%}$
13.6 TeV	σ^{LO} [fb]	$70.59(2)^{+26.3\%}_{-19.7\%}$	$70.6^{+26\%}_{-20\%}$
	σ^{NLO} [fb]	$130.50(9)^{+16.3\%}_{-13.8\%}$	$130.5^{+16\%}_{-14\%}$
14 TeV	σ^{LO} [fb]	$75.15(2)^{+26.0\%}_{-19.6\%}$	$75.2^{+26\%}_{-20\%}$
	σ^{NLO} [fb]	$139.0(1)^{+16.3\%}_{-13.7\%}$	$138.9^{+16\%}_{-14\%}$

Table 1: Comparison with results of Ref. [35] at $\sqrt{s} = \{13, 13.6, 14\}$ TeV. Tables taken from Ref. [2].

including different number of terms in the mass expansion. We find that both expansions agree well with each other typically in the phase-space region around $p_T = 200$ GeV, where both methods can be safely used for precise results. Towards smaller p_T values only the $t \to 0$ expansion leads to reliable results while the high-energy expansion diverges, For high p_T, the behaviour is reversed, and only the high-energy expansion is accurate. In Figure 1 we show the real and imaginary parts of the helicity amplitude $\mathcal{A}^{(1)}_{++++}$ for the $gg \to Z^\star Z^\star$ process as a function of $\sqrt{s}$ for $p_T = 150$ GeV. We find that both expansion methods lead to essentially the same results for all possible values of $\sqrt{s}$. Therefore, the region of convergence of both methods is limited only by the value of p_T, which is then used to determine which expansion method should be used. Note that the convergence of the high-energy expansion is additionally limited to the region above the top-quark pair threshold, $\sqrt{s} > 2m_t \sim 345$ GeV. In practise, such small values are never used for the high-energy expansion due to the lower limit on p_T.

4. Hadronic cross sections with ggxy

We have implemented the results for $gg \to ZH$ into our C++ library ggxy[1] which can be used for the calculation of the one- and two-loop amplitudes as well as the hadronic cross section at NLO QCD. Furthermore, ggxy is also used to provide a process implementation[2] of $gg \to \ell^-\ell^+H$ and $gg \to \nu_\ell\bar{\nu}_\ell H$ in POWHEG, which makes the matching to parton showers possible.

The one-loop $2 \to 3$ amplitudes required for the real corrections are computed with the tools Recola [26], Collier [27], CutTools [28] and OneLOop [29]. The calculation of the real corrections is organized with the Catani-Seymour dipole subtraction scheme [30] and the phase-space integration is performed with avhlib [31, 32]. In addition, the conversion of the on-shell top-quark mass to the one in the $\overline{\text{MS}}$ scheme and the running of it is obtained with CRunDec [33]. For the numerical evaluation of polylogarithmic functions we use the code of Ref. [34].

We have performed several cross-checks at the integrated and differential level with existing results to ensure the correctness of our implementation of $gg \to ZH$ at NLO QCD. An example of these cross-checks is shown in Table 1, where we compare the integrated cross section at LO

[1] https://gitlab.com/ggxy/ggxy-release
[2] https://gitlab.com/POWHEG-BOX/V2/User-Processes/ggxy_ggZH

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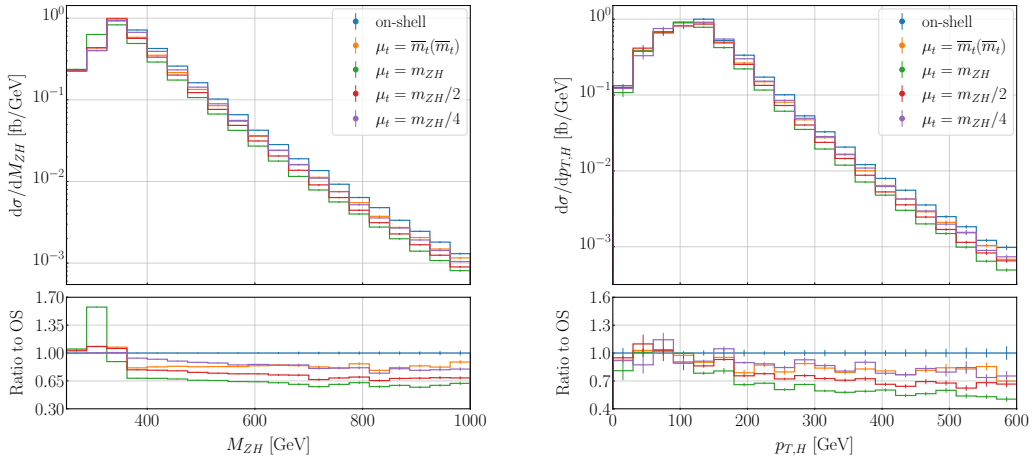

Figure 3: Dependence on the top quark mass renormalization scheme for the distributions M_{ZH} and $p_{T,H}$. Results are shown in the on-shell and $\overline{\text{MS}}$ top-quark mass renormalization schemes for different values of the top-quark mass renormalization scale μ_t. Lower panels show the ratio to the on-shell scheme. Figures taken from Ref. [2].

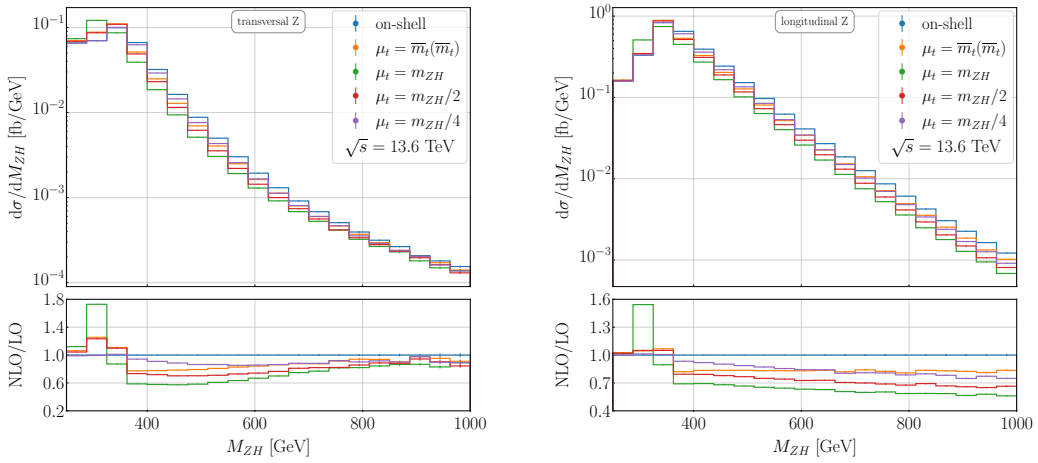

Figure 4: Same as Figure 3 but for a transversely and longitudinally polarized Z boson for the M_{ZH} distribution.

and NLO QCD for several center-of-mass energies with the results of Ref. [35]. Both computations agree well within in the statistical uncertainties. A runtime of about 90 minutes is required to obtain a precision of 0.1% – 0.2% on a single core, which makes the computation of $gg \to ZH$ roughly three times slower than the one of $gg \to HH$.

In Figure 3 we show a comparison for the distributions M_{ZH} and $p_{T,H}$ between the on-shell and $\overline{\text{MS}}$ top-quark mass renormalization schemes at NLO QCD, where in the latter cases we use different values for the top-quark mass renormalization scale μ_t. We find differences between these predictions up to 30% – 40%, which are similar size as the top-quark mass scheme differences for $gg \to HH$, see e.g. Ref. [1]. To better understand the origin of these differences, we also study the top-quark mass scheme dependence for a polarized Z boson. In Figure 4 we show the same

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comparison for a transversely and longitudinally polarized Z boson for the M_{ZH} distribution. We find similar large differences between the different predictions for the longitudinal polarization of the Z boson, while the differences in the case of a transversely polarized Z boson decrease towards the high-energy region. In addition, the contribution from the transversal polarization is suppressed by almost an order of magnitude, so that the effects for the unpolarized cross section is mainly coming from the longituidinal polarization of the Z boson.

5. Conclusions and outlook

In these proceedings we have presented the computation of the two-loop amplitudes for $gg \to ZH$ and $gg \to Z^\star Z^\star$ with full top-quark mass dependence using two different expansion methods that combined cover the full phase-space. The latter results can directly be reused for the $gg \to Z^\star\gamma^\star$ and $gg \to \gamma^\star\gamma^\star$ processes. The results of $gg \to ZH$ have been implemented in the C++ library `ggxy` that can be used for the efficient and numerically stable evaluation of the two-loop amplitudes as well as the computation of the hadronic cross sections at NLO QCD. In addition to unpolarized hadronic cross sections, we have implemented the capability to compute hadronic cross section with a transversely or longitudinally polarized Z boson. Furthermore, these results have been used for the implementation of $gg \to \ell^-\ell^+H$ and $gg \to \nu_\ell\bar{\nu}_\ell H$ at NLO QCD in `POWHEG`.

The extension of polarized $gg \to ZH$ production as well as the implementation of the amplitudes for $gg \to VV$ production will be available in a future version of `ggxy`.

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