

## Pseudoscalar Singlet Physics with Staggered Fermions

---

**Eric B. Gregory\***, Alan C. Irving, Craig McNeile, Steven Miller, and Zbigniew Sroczynski

*Theoretical Physics Division*

*Department of Mathematical Sciences*

*University of Liverpool*

*Liverpool, L69 7ZL, UK*

*E-mail: [egregory@liverpool.ac.uk](mailto:egregory@liverpool.ac.uk)*

We report on progress in measuring disconnected correlators associated with pseudoscalar flavor-singlet mesons. This will eventually allow us to compute the masses of the eta and eta' mesons. Flavor-singlet physics also presents an interesting test of the staggered fermion formulation, as disconnected correlators are sensitive to whether the same action governs both sea quarks and valence quarks. It can also help test the validity of the “fourth-root trick” used in unquenched lattice calculations where the number of flavors  $N_f < 4$ .

*XXIIIrd International Symposium on Lattice Field Theory*

*25-30 July 2005*

*Trinity College, Dublin, Ireland*

---

\*Speaker.

## 1. Introduction

Pseudoscalar flavor-singlet mesons are interesting for a number of reasons, including the relation between the  $\eta'$  and  $\pi$  mass difference and the U(1) axial anomaly [1, 2], and the connection to the topological susceptibility. Unlike flavor non-singlet states, the propagator of flavor-singlet meson contains disconnected correlators, which are particularly challenging to measure precisely in lattice simulations. Hence flavor-singlet mesons present a rich proving ground for lattice simulations. To this end a number of simulations have been performed on the lattice using quenched Wilson [3],  $N_f = 2$  Wilson [4, 5, 6, 7, 8], and  $N_f = 2$  staggered fermion formulations [9, 10].

Simulations utilizing modern dynamical staggered fermion gauge configurations are of particular interest. The library of  $N_f = 2 + 1$  staggered fermion gauge configurations from the MILC collaboration [12, 13] contains extremely light up and down quarks ( $m_q/m_s < 0.2$ ), simulated at fine lattice spacings ( $a \approx 0.11\text{fm}$  and  $a \approx 0.09\text{fm}$ ). These configurations, and other ensembles currently being generated provide an unprecedented opportunity to measure mixed operators numerically determine the masses of both the  $\eta$  and  $\eta'$ . Additionally, the sensitivity of disconnected correlators to sea quark loops offers a probe of the validity of the fourth-root trick in the staggered fermion formulation.

With this in mind, we make a preliminary presentation of measurements of pseudoscalar flavor-singlet propagators on  $N_f = 2 + 1$ -flavor improved staggered fermion configurations. As the work is in preliminary stages, we concentrate on some theoretical and technical issues in Sections 2 and 3. In Section 4 we describe our early results with some discussion of ongoing questions and issues, which we hope our continuing work will resolve.

## 2. Theoretical Considerations

The flavor-singlet pseudoscalar meson propagator is expressed as

$$G(x', x) = \langle \sum_i \bar{q}_i(x') (\gamma_5 \otimes \mathbf{1}) q_i(x') \sum_j \bar{q}_j(x) (\gamma_5 \otimes \mathbf{1}) q_j(x) \rangle, \quad (2.1)$$

where the  $(\gamma_5 \otimes \mathbf{1})$  denotes the meson as a pseudoscalar in spinor-space and a singlet in flavor-space. One can group the possible contractions into two classes:  $N_f$  connected terms with contractions connecting fields at  $x$  and  $x'$ , and  $N_f^2$  disconnected terms with the  $q$ s contracted with  $\bar{q}$ s at the same space time points. So the full propagator is:

$$G = N_f C - N_f^2 D, \quad (2.2)$$

where  $C$  and  $D$  are the connected and disconnected correlators, respectively. The minus sign is due to the additional fermion loop in  $D$ . The function  $C$  is also the connected pion propagator with  $(\gamma_5 \otimes \mathbf{1})$  operator. In full QCD, both  $C$  and  $G$  will have leading behavior that decays exponentially:

$$C(t) \sim e^{-m_\pi t} \quad \text{and} \quad G(t) \sim e^{-m_{\eta'} t}. \quad (2.3)$$

So the ratio of the disconnected to connected contributions to the singlet propagator behaves as

$$R(t) \equiv \frac{N_f^2 D(t)}{N_f C(t)} = \frac{N_f C(t) - G(t)}{N_f C(t)} = 1 - A \exp[-(m_{\eta'} - m_\pi)t] \quad (2.4)$$

in full QCD [9]. The correct behavior of the full propagator depends on the sea quarks having the same number of flavors and same masses as the valence quarks. If the  $\det^{1/4}$  trick introduces some unexpected pathology into the action of the sea quarks, we might expect to detect a deviation from the form of equation 2.4. For example, in the limit of quenched QCD  $R$  should be linear:

$$R(t) = A + Bt. \quad (2.5)$$

In non-degenerate flavor simulations one should employ obvious generalizations of equations 2.2 and 2.4. For example, when  $N_f$  is split into  $N_q + N_s$  flavors of light and strange quarks we have:

$$\begin{aligned} G &\longrightarrow (N_q C_{qq} + N_s C_{ss}) - (N_q^2 D_{qq} + N_s^2 D_{ss} + 2N_q N_s D_{qs}) \\ R &\longrightarrow \frac{N_q^2 D_{qq} + N_s^2 D_{ss} + 2N_q N_s D_{qs}}{N_q C_{qq} + N_s C_{ss}}. \end{aligned} \quad (2.6)$$

With staggered fermions, one must rescale the disconnected correlators by  $1/4$  with respect to the connected correlators to take account the two loops of four staggered valence tastes in the disconnected diagrams compared to the single valence loop in the connected diagrams[9]. In the discussion below such rescaling is implicit in  $D$ .

### 3. Simulation and Measurement

We have begun measurement of singlet propagators on the following gauge ensembles:

$N_f$	$\beta$	$L^3 \times T$	$am$	$N_{\text{configs}}$
0	8.00	$16^3 \times 32$	0.02	104
2	7.20	$16^3 \times 32$	0.02	268
2+1	6.76	$20^3 \times 64$	0.007, 0.05	422
2+1	6.76	$20^3 \times 64$	0.01, 0.05	644

The  $16^3 \times 32$  configurations are small test lattices generated locally, while the  $20^3 \times 64$  configurations are part of the library of MILC ‘‘coarse lattices’’ [12].

In principle there are two choices of flavor-singlet pseudoscalar meson operator available. These are the  $(\gamma_4 \gamma_5 \otimes \mathbf{1})$  and the  $(\gamma_5 \otimes \mathbf{1})$ . The former is a three-link operator, with the quark and antiquark sources set on opposite corners of the spatial cube, while the latter is a four-link operator, with the quark and antiquark situated on opposite corners of the *hypercube*. The  $(\gamma_4 \gamma_5 \otimes \mathbf{1})$  has a parity partner, namely the scalar  $(\mathbf{1} \otimes \gamma_4 \gamma_5)$ , which contributes an oscillating exponential to the pseudoscalar propagator. The parity partner of the  $(\gamma_5 \otimes \mathbf{1})$ , however, has exotic quantum numbers  $J^{PC} = 0^{+-}$ , and contributes nothing to the pseudoscalar propagator. Hence, only the  $(\gamma_5 \otimes \mathbf{1})$  is well-suited for looking at quantities such as the ratio in equation 2.4. We formulate the operators in a gauge invariant way, using symmetric covariant shifts to displace the quark and antiquark sources to their respective positions on the hypercube.

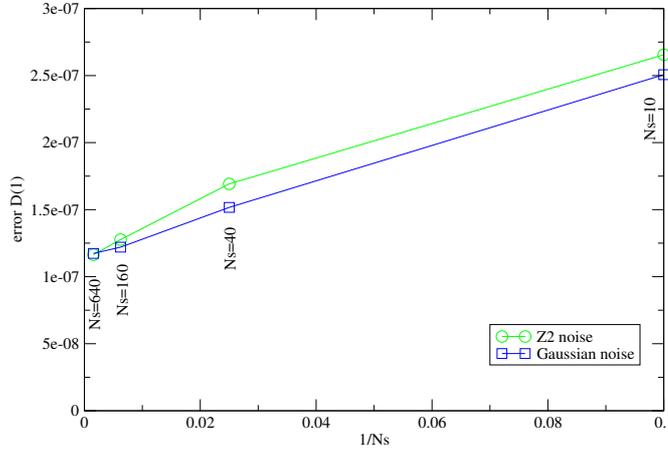
We measure the connected diagrams using standard point sources. We measure the the disconnected diagrams with a stochastic volume source method [14]. We define a source field  $\eta$  whose value at every lattice site is drawn from a gaussian distribution. Then we solve for

$$\mathcal{O}(t) = \frac{1}{N_{\text{src}}} \sum_i^{N_{\text{src}}} \sum_{x, x_4=t} \sum_{y, y_4=t} \text{Tr} \eta_y^{\dagger(i)} \Delta_{\gamma_5 \otimes \mathbf{1}} M_{yx}^{-1} \eta_x^{(i)}, \quad (3.1)$$

where  $M$  is the fermionic matrix and  $\Delta_{\gamma_r \otimes \mathbf{1}}$  is the staggered meson operator that effects the four-link shifts and the Kogut-Suskind phasing appropriate to the  $\gamma_r \otimes \mathbf{1}$  meson. We average over a number of different source fields  $\eta$ , then compute the disconnected correlator:

$$D(\Delta t) = \langle \mathcal{O}(t) \mathcal{O}(t + \Delta t) \rangle \quad (3.2)$$

In tests on the small ( $16^3 \times 32$ ) lattices, we found that with as few as 40 noise sources our error would be dominated by gauge noise even with several hundred configurations, so in subsequent runs we used  $N_{\text{src}} = 40$ . We also tested  $Z_2$  noise and found that it produced noise errors that were systematically larger than that obtained with gaussian noise, as shown in Figure 1. Previously, for Wilson fermions  $Z_2$  noise was found to be better than Gaussian for the simplest stochastic estimators [16].



**Figure 1:** Error on the  $D(t = 1)$  for  $N_f = 2$   $\beta = 7.2$   $am = 0.20$  on  $16^3 \times 32$  lattices for gaussian and  $Z_2$  noise as a function of the inverse number of sources.

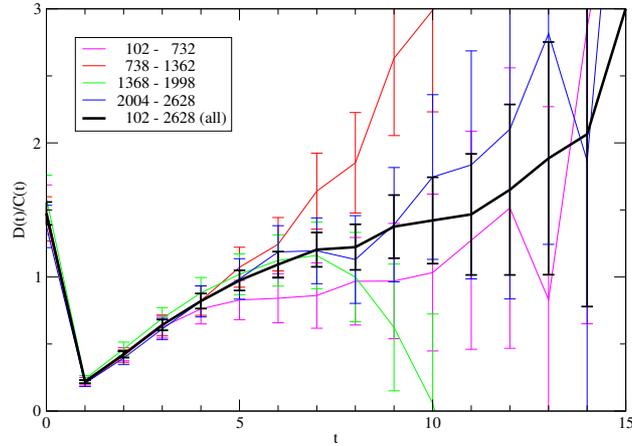
#### 4. Results and Discussion

Our results to date show clear signals for disconnected and connected correlators. For the 2+1-flavor ensembles, we formed the ratio:

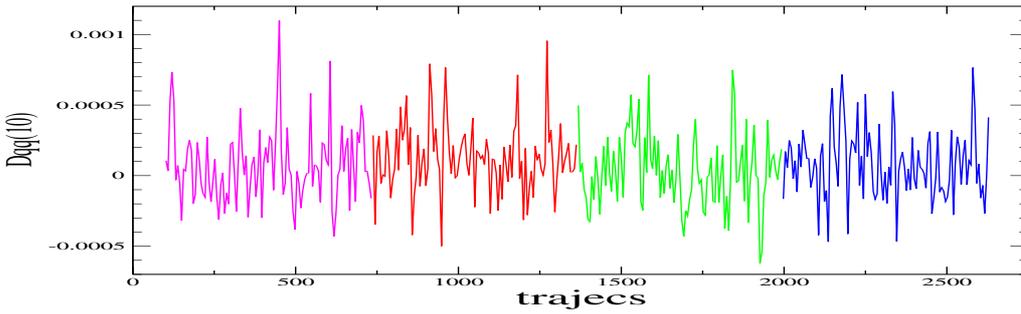
$$R(t) = \frac{4D_{qq}(t) + 4D_{qs}(t) + 1D_{ss}}{2C_{qq}(t) + C_{ss}(t)}. \quad (4.1)$$

An example of the resulting function is plotted as the dark curve in Figure 2. Results for both 2 + 1-flavor ensembles were qualitatively similar.

It is clear that our correlators as formulated do not form the components of a singlet propagator which is positive at all  $t$ , as the magnitude of the disconnected correlators exceeds that of the connected correlators for  $t > 5$ . Correctly normalized correlators showing this behavior might



**Figure 2:**  $R(t)$  on  $\beta = 6.76$ ,  $am = 0.007, 0.05$  on  $20^3 \times 64$  lattices for full time series (black), and four subsets (colored). Configurations indexed by trajectory number (6 trajectories per configuration).



**Figure 3:** Time series for  $D_{qq}(t = 10)$  on  $\beta = 6.76$ ,  $am = 0.007, 0.05$  on  $20^3 \times 64$  lattices. Same color scheme as previous figure.

illustrate a flaw in the simulated staggered sea. We have suspicions that the normalizations of our disconnected operators may instead be incorrect, however. As we compute the disconnected and connected correlators by different methods, we must account for different numerical factors in each, and reconciling them is non-trivial. Further tests are being made.

Further uncertainty arises from long autocorrelations in the disconnected correlators. Figure 2 also shows the ratio plotted with four different subsets of the ensemble, each composed of a quarter of the time series. The large difference between bins suggests that one needs far more gauge configurations to make a statement about the behavior of the  $D$  to  $C$  ratio. The autocorrelations are not visually apparent from inspection of the time series of the disconnected correlators, e.g. Figure 3, but spikes in otherwise small fluctuations seem to be enough to strongly affect  $D/C$ . It is also possible that with a longer time series  $R(t)$  may settle at a smaller asymptote than is apparent now.

With these uncertainties, we do not conclude that the pseudoscalar singlet correlators produced thus far cast doubt on the staggered formulation, nor are we at this stage able to make statements about the lattice masses of the  $\eta$  and  $\eta'$ . It is evident that we will first have to confirm the correct normalizations, and then make measurements on extremely long time series ensembles. We are in the process of generating  $2+1$ -flavor ensembles of  $\sim 10^4$  configurations on the UKQCD's QCDOC machine. Additionally, we are investigating different optimization strategies [9, 15] such as dilution of the stochastic sources which may decrease disconnected correlator noise.

## References

- [1] E. Witten, *Current Algebra Theorems For The  $U(1)$  'Goldstone Boson'*, *Nucl. Phys. B* **156** (269).
- [2] G. Veneziano,  *$U(1)$  Without Instantons*, *Nucl. Phys. B* **159** (213).
- [3] S. Itoh, Y. Iwasaki and T. Yoshie, *The  $U(1)$  Problem And Topological Excitations On A Lattice*, *Phys. Rev. D* **36**, (527).
- [4] C. McNeile and C. Michael [UKQCD Collaboration], *The eta and eta' mesons in QCD*, *Phys. Lett. B* **491** (123) [Erratum-ibid. B **551**, 391 (2003)] [arXiv:hep-lat/0006020].
- [5] T. Struckmann *et al.* [TXL Collaboration], *Flavor singlet pseudoscalar masses in  $N(f) = 2$  QCD*, *Phys. Rev. D* **63** (074503) [arXiv:hep-lat/0010005].
- [6] V. I. Lesk *et al.* [CP-PACS Collaboration], *Flavor singlet meson mass in the continuum limit in two-flavor lattice QCD*, *Phys. Rev. D* **67** (074503) [arXiv:hep-lat/0211040].
- [7] K. Schilling, H. Neff and T. Lippert, *Computing the eta and eta' mesons in lattice QCD*, [arXiv:hep-lat/0401005].
- [8] C. R. Allton *et al.* [UKQCD Collaboration], *Improved Wilson QCD simulations with light quark masses*, *Phys. Rev. D* **70** (014501) [arXiv:hep-lat/0403007].
- [9] L. Venkataraman and G. Kilcup, *The eta' meson with staggered fermions*, [arXiv:hep-lat/9711006].
- [10] J. B. Kogut, J. F. Lagae and D. K. Sinclair, *Topology, fermionic zero modes and flavor singlet correlators in finite temperature QCD*, *Phys. Rev. D* **58** (054504) [arXiv:hep-lat/9801020].
- [11] H. Fukaya and T. Onogi, *theta vacuum effects on the chiral condensation and the eta' meson correlators in the two-flavor massive QED(2) on the lattice*, *Phys. Rev. D* **70**, (054508)[arXiv:hep-lat/0403024].
- [12] C. W. Bernard *et al.*, *The QCD spectrum with three quark flavors*, *Phys. Rev. D* **64**, (054506) [arXiv:hep-lat/0104002].
- [13] C. Aubin *et al.*, *Light hadrons with improved staggered quarks: Approaching the continuum limit*, *Phys. Rev. D* **70** (094505) [arXiv:hep-lat/0402030].
- [14] F. Farchioni, G. Muenster and R. Peetz, *The volume source technique for flavor singlets: A second look*, *Eur. Phys. J. C*, **38** (329) [arXiv:hep-lat/0404004].
- [15] J. Foley, K. Jimmy Juge, A. O'Cais, M. Peardon, S. M. Ryan and J. I. Skullerud, *Practical all-to-all propagators for lattice QCD*, [arXiv:hep-lat/0505023].
- [16] S. J. Dong and K. F. Liu, *Stochastic estimation with  $Z(2)$  noise*, *Phys. Lett. B* **328** (130) [arXiv:hep-lat/9308015].