

The Status of D-Theory

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Field theories are usually quantized by performing a path integral over configurations of classical fields. This is the case both in perturbation theory and in Wilson's nonperturbative lattice field theory. *D-theory* is an alternative nonperturbative formulation of field theory in which classical fields emerge from the low-energy collective dynamics of *discrete* quantum variables (quantum spins and their gauge analogs — quantum links) which undergo *dimensional* reduction. D-theory was developed some time ago as a discrete approach to $U(1)$ and $SU(2)$ pure gauge theories [1], extended to $SU(N)$ gauge theories and full QCD in [2, 3], and also applied to a variety of other models [4, 5]. On the practical side, D-theory provides a framework for the development of efficient numerical methods, such as cluster algorithms. For example, in the D-theory formulation of $CP(N-1)$ models one can simulate efficiently at non-zero chemical potential [6] or at non-zero vacuum angle θ [7]. On the conceptual side, D-theory offers a natural solution for the nonperturbative hierarchy problem of chiral symmetry in QCD. We also take a broader nonperturbative view on fundamental physics and speculate that D-theory variables — i.e. quantum spins and quantum links — may be promising candidates for the physical degrees of freedom that Nature has chosen to regularize the standard model physics at ultra-short distances.

XXIIIrd International Symposium on Lattice Field Theory

25-30 July 2005

Trinity College, Dublin, Ireland

*Speaker.

†This work is supported by the Schweizerischer Nationalfonds.

1. The D-Theory Formulation of $CP(N-1)$ Models

To illustrate D-theory in a simple setting, let us consider 2-d $CP(N-1)$ models [8]. Just like 4-d QCD, these models are asymptotically free, they have a nonperturbatively generated massgap, as well as instantons and hence θ -vacua. Let us imagine a toy “world” whose “standard model” is just a $CP(N-1)$ model with the Euclidean action

$$S[P] = \int d^2x \frac{1}{g^2} \text{Tr}[\partial_\mu P \partial_\mu P] - i\theta Q[P]. \quad (1.1)$$

Here $P(x) \in CP(N-1) = SU(N)/U(N-1)$ is a Hermitean $N \times N$ matrix-valued field which obeys $P(x)^2 = P(x)$, $P(x)^\dagger = P(x)$, and $\text{Tr}P(x) = 1$. Furthermore, g is the coupling constant and $\theta \in [-\pi, \pi]$ is the vacuum angle which multiplies the topological charge

$$Q[P] = \frac{1}{2\pi i} \int d^2x \varepsilon_{\mu\nu} \text{Tr}[P \partial_\mu P \partial_\nu P] \in \Pi_2[SU(N)/U(N-1)] = \Pi_1[U(N-1)] = \Pi_1[U(1)] = \mathbf{Z}. \quad (1.2)$$

The model has a global $SU(N)$ symmetry $P(x)' = \Omega P(x) \Omega^\dagger$, with $\Omega \in SU(N)$.

Let us assume that the actual physical values of the parameters in the toy world are $N = 3$ and $\theta = 0$. For the fun of the argument (and happily ignoring the anthropic principle) let us further pretend that there are toy world physicists just as puzzled about the ultimate short distance physics of their world as we are about our own. Our $(1+1)$ -d colleagues would be quick to figure out that their standard model is asymptotically free [8], thus solving their “hierarchy problem” of why the low-energy physics takes place so far below the ultimate cut-off. Still, the toy world’s physics community would remain puzzled about their “strong CP problem”: Why is $\theta = 0$? The $(1+1)$ -d physicists would be able to explain “confinement”, i.e. the absence of massless excitations, as a consequence of the Hohenberg-Mermin-Wagner-Coleman theorem. At large N they could analytically calculate the θ -dependence and find a first order phase transition at $\theta = \pm\pi$ [9]. At finite $N \geq 3$, on the other hand, they would be unable to calculate the θ -dependence or the massgap analytically. At this point, some toy world physicist may come up with the idea to regularize the theory on the lattice. However, not unlike in our own world, Wilson’s lattice field theory faces severe algorithmic problems. For example, one can show that Wolff-type embedding cluster algorithms do not work efficiently for $CP(N-1)$ models with $N \geq 3$ [10]. Multigrid methods work reasonably well, but only at $\theta = 0$ [11].

D-theory offers an alternative regularization that allows one to make substantial algorithmic progress. In the case of $CP(N-1)$ models, the discrete D-theory variables are generalized quantum spins $T_x^a = \frac{1}{2} \lambda_x^a$ which generate an $SU(N)$ symmetry $[T_x^a, T_y^b] = i \delta_{xy} f_{abc} T_x^c$. The spins are located on the sites x of a square lattice with spacing a of size $L \times L'$, with $L \gg L'$ and with periodic boundary conditions. Hence, as shown in figure 1, we are dealing with a quantum spin ladder consisting of $n = L'/a$ transversely coupled spin chains of length L . The x -direction of size L corresponds to the spatial dimension of the target $CP(N-1)$ model, while the extra y -dimension of finite extent L' will ultimately disappear via dimensional reduction. We consider nearest-neighbor couplings which are antiferromagnetic along the chains and ferromagnetic between different chains. Hence, the lattice decomposes into two sublattices A and B with even and odd sites along the x -direction, respectively. The spins T_x^a on sublattice A transform in the fundamental representation $\{N\}$ of $SU(N)$, while the

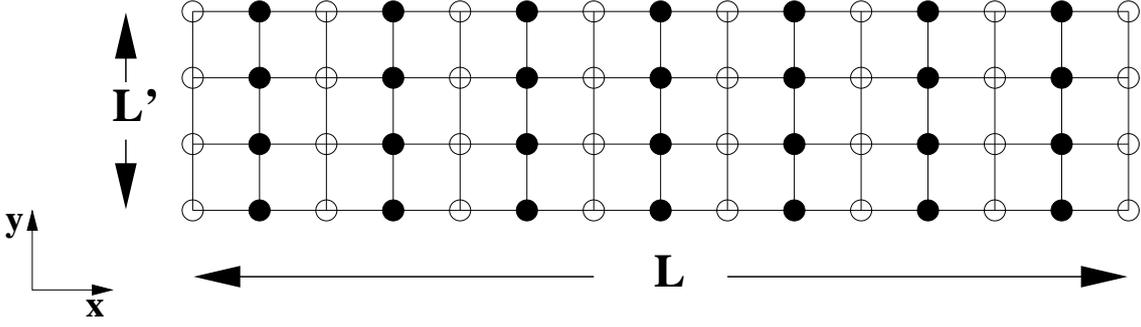


Figure 1: Spin ladder with two sublattices A (open circles) and B (filled circles).

ones on sublattice B are in the anti-fundamental representation $\{\bar{N}\}$ and are thus described by the conjugate generators $-T_x^{a*}$. The quantum spin ladder Hamiltonian is given by

$$H = -J \sum_{x \in A} [T_x^a T_{x+\hat{1}}^{a*} + T_x^a T_{x+\hat{2}}^a] - J \sum_{x \in B} [T_x^{a*} T_{x+\hat{1}}^a + T_x^{a*} T_{x+\hat{2}}^{a*}], \quad (1.3)$$

where $J > 0$, and $\hat{1}$ and $\hat{2}$ are unit-vectors in the spatial x - and y -directions, respectively. By construction the system has a global $SU(N)$ symmetry, i.e. $[H, T^a] = 0$, with the total spin given by $T^a = \sum_{x \in A} T_x^a - \sum_{x \in B} T_x^{a*}$.

One finds that, at zero temperature, the infinite system (with both $L, L' \rightarrow \infty$) undergoes spontaneous symmetry breaking from $SU(N)$ to $U(N-1)$. Hence, there are massless Goldstone bosons (spin waves) described by fields in the coset space $SU(N)/U(N-1) = CP(N-1)$. Using chiral perturbation theory, the lowest-order terms in the Euclidean effective action for the spin waves are given by

$$S[P] = \int_0^\beta dt \int_0^L dx \int_0^{L'} dy \text{Tr} \{ \rho'_s \partial_y P \partial_y P + \rho_s [\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P] - \frac{1}{a} P \partial_x P \partial_t P \}. \quad (1.4)$$

Here $\beta = 1/T$ is the inverse temperature, ρ_s and ρ'_s are spin stiffness parameters for the x - and y -direction, respectively, and c is the spin wave velocity. The last term in the integrand of eq.(1.4) is purely imaginary and is related to the topological charge $Q[P]$, which is a y -independent integer. Hence, the y -integration in the last term of eq.(1.4) can be performed trivially. This yields $i\theta Q[P]$ where the vacuum angle is given by $\theta = L'\pi/a = n\pi$. Here a is the lattice spacing of the quantum spin ladder and $L'/a = n$ is the number of transversely coupled spin chains. Hence, for even n the vacuum angle is trivial, and for odd n it corresponds to $\theta = \pi$.

While the infinite $(2+1)$ -d system has massless Goldstone bosons, the Coleman-Hohenberg-Mermin-Wagner theorem forbids the existence of massless excitations once the y -direction is compactified to a finite extent L' . As a consequence, the Goldstone bosons then pick up a nonperturbatively generated massgap $m = 1/\xi$ and thus have a finite correlation length ξ . Interestingly, for sufficiently many transversely coupled chains, the correlation length becomes exponentially large $\xi \propto \exp(4\pi L' \rho_s / cN) \gg L'$, and the system undergoes dimensional reduction to the $(1+1)$ -d $CP(N-1)$ field theory with the action

$$S[P] = \int_0^\beta dt \int_0^L dx \text{Tr} \{ \frac{1}{g^2} [\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P] - n P \partial_x P \partial_t P \}. \quad (1.5)$$

The coupling constant of the dimensionally reduced theory is given by $1/g^2 = L'\rho_s/c$. This type of dimensional reduction is well-known for antiferromagnets [12, 13].

When regularized using D-theory a highly efficient loop-cluster algorithm can be applied to $CP(N-1)$ models [7]. In this way large correlation lengths of up to 250 lattice spacings have been simulated with no indication of critical slowing down. It has also been possible to simulate at non-zero chemical potential [6] and at vacuum angle $\theta = \pi$ [7]. In this way, it was shown for several $N \geq 3$ that there is a first order phase transition at $\theta = \pi$ at which charge conjugation gets spontaneously broken. Algorithmic developments for the D-theory formulation of QCD are currently under intensive investigation. Ironically, while the meron-cluster algorithm provides a very efficient method to simulate dynamical fermions [14], at present the simulation of quantum links still causes severe problems.

When the experimentalists in our toy world will be able to probe the shortest distance scales, they may discover the (perhaps somewhat disappointing) fact that they actually live on, let us say, $n = 10$ transversely coupled chains of $SU(3)$ spins. This also solves their “strong CP-problem”: $\theta = 0$ because n is even. In the following, we like to speculate that our own world may in some respects be not so different from the toy example.

2. A Nonperturbative View on Fundamental Physics

Dimensional regularization provides an elegant but unphysical regularization, which is very useful in QCD, but it defines the theory only in perturbation theory. In a chiral gauge theory like the full standard model, dimensional regularization of γ_5 is subtle beyond one loop. Such subtleties provide a first perturbative glance at a deep problem that becomes apparent when one regularizes theories with a chiral symmetry beyond perturbation theory. In Wilson’s lattice field theory, due to fermion doubling, chiral symmetry has posed severe problems for many years. In particular, using Wilson fermions, i.e. removing the doubler fermions by breaking chiral symmetry explicitly, causes a severe nonperturbative hierarchy problem for fermions [15]. Without unnatural fine-tuning of the bare fermion mass it is then impossible to obtain light fermions. This problem has sometimes been viewed as a deficiency of the lattice regularization. In particular, the global chiral symmetry of massless QCD is usually taken for granted because it can easily be maintained in continuum regularization schemes. A continuum field theorist could “explain” the presence of (almost) massless fermions in Nature by the existence of (an approximate) chiral symmetry which protects the quark masses from running to the cut-off scale. We like to stress that this perturbative point of view of the problem is rather limited, and may even prevent us from drawing some far-reaching conclusions about the physics at ultra-short distance scales.

Remarkably, the hierarchy problem of the nonperturbative regularization of chiral symmetry has found an elegant solution in terms of Kaplan’s domain wall fermions [16, 17]. Massless 4-d fermions then arise naturally as states localized on a domain wall embedded in a 5-d space-time, while fermion doublers are still removed by a 5-d Wilson term. Narayanan’s and Neuberger’s closely related overlap fermions [18] are also deeply related to the physics of an extra dimension. Hence, solving the nonperturbative hierarchy problem of fermions may require at least one extra dimension. Without invoking extra dimensions, at a nonperturbative level we can presently not understand how fermions can be naturally light. The existence of light fermions in Nature may

thus be a concrete hint to the physical reality of extra dimensions. In particular, we don't need string theory or other physics beyond the standard model to motivate extra dimensions. The mere existence of light fermions in Nature is evidence already. This important hint from nonperturbative physics is indeed easily missed when one considers chiral symmetry only in a perturbative context.

Why is the weak scale so much smaller than the GUT or Planck scale? This is the gauge hierarchy problem of the standard model. In contrast to the nonperturbative hierarchy problem of chiral symmetry, the gauge hierarchy problem manifests itself already in perturbation theory and is therefore widely appreciated. The presently most popular potential solution of this problem relies on supersymmetry. However, from a nonperturbative point of view this "solution" is not yet satisfactory. Beyond perturbation theory, namely on the lattice, a priori supersymmetry is as undefined as chiral symmetry was before Kaplan constructed lattice domain wall fermions. Lattice scalar field theory suffers from the same hierarchy problem as the continuum theory, i.e. without unnatural fine-tuning of the bare mass, the vacuum value of the Higgs field remains at the lattice cut-off. Obtaining supersymmetry in the continuum limit thus requires fine-tuning of the bare scalar mass. Of course, as long as the nonperturbative construction of supersymmetry itself requires unnatural fine-tuning, it cannot solve the hierarchy problem. In the worst case, the supersymmetric extension of the standard model may just be a perturbative illusion which does not arise naturally, i.e. without fine-tuning, in a nonperturbative context. Until now, other than for chiral symmetry, Nature has not yet provided us with experimental evidence for supersymmetry. While this may well change in the near future, one can presently not be sure that supersymmetric extensions of the standard model even exist naturally beyond perturbation theory.

Ultimately, the divergences of quantum field theory imply that the concept of a classical field (originally developed for classical electrodynamics) breaks down at ultra-short distances. In particular, Dirac continued to point out that he was unsatisfied with the formal procedures of removing singularities in the perturbative treatment of QED [19]. Indeed, it is hard to imagine that classical fields are the truly fundamental physical degrees of freedom that Nature has chosen to regularize particle physics. No matter if there are strings, branes, or some tiny wheels turning around at the Planck scale, Nature must have found a concrete way to regularize gravity as well as the standard model physics at ultra-short distances. Of course, the identification of the ultimate hardware on which the basic laws of Nature are implemented is a very difficult task which may or may not be within reach of physics in the foreseeable future. Here we like to speculate that discrete variables, namely quantum spins and their gauge analogs —quantum links— may be promising candidates for Nature's most fundamental degrees of freedom.

D-theory provides a framework in which the familiar classical fields emerge naturally from discrete quantum variables that undergo dimensional reduction. In the D-theory formulation of QCD [2] a fifth dimension is not only needed to obtain naturally light quarks, but also to assemble 4-d gluons out of 5-d quantum links. If we take the existence of light fermions as a hint to the reality of extra dimensions, we should take these dimensions seriously also for the gauge fields. Just like Wilson's parallel transporters, quantum links are $N \times N$ matrices which transform appropriately under $SU(N)$ gauge transformations. However, like the components of a quantum spin, their matrix elements are operators (in the QCD case generators of $SU(2N)$). The collective dynamics of the discrete quantum link variables may give rise to a 5-d non-Abelian Coulomb phase which is analogous to the 3-d phase with massless Goldstone bosons. Just as Goldstone bosons

pick up a mass as a consequence of the Hohenberg-Mermin-Wagner-Coleman theorem when the third direction is compactified, with a compact fifth dimension Coulombic gluons form glueballs and are thus confined [1].

Until now D-theory has not been widely recognized as a potential framework for a truly fundamental theory. Although this is highly speculative, we like to point out that D-theory indeed offers room for nonperturbative thought on fundamental physics alternative to string theory. Of course, the present constructions with a rigid lattice and just one extra dimension may not be sufficient, but the idea that the most fundamental degrees of freedom are discrete quantum variables may lead to fruitful developments. Regularizing the full standard model or gravity in the D-theory framework represent great challenges that seem worth facing.

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