PROCEEDINGS OF SCIENCE

The nucleon axial charge and lowest moment $\langle x \rangle$ with Nf = 2 dynamical twisted mass fermions

Rémi Baron[*,†], **Pierre Guichon**
CEA, Centre de Saclay, IRFU/Service de Physique Nucléaire, F-91191 Gif-sur-Yvette, France
E-mail: remi.baron@cea.fr, pierre.guichon@cea.fr

Mariane Brinet, Jaume Carbonell, Vincent Drach, Pierre-Antoine Harraud
Laboratoire de Physique Subatomique et Cosmologie, 53 Av des martyrs, 38026 Grenoble,France
E-mail: mariane@lpsc.in2p3.fr, carbonel@lpsc.in2p3.fr, drach@lpsc.in2p3.fr, harraud@lpsc.in2p3.fr

Zhaofeng Liu, Olivier Pène
Laboratoire de Physique Théorique, UMR8627, Université Paris XI, 91405 Orsay-Cedex, France
E-mail: zhaofeng.liu@th.u-psud.fr, Olivier.Pene@th.u-psud.fr

Vincent Morenas
Laboratoire de Physique Corpusculaire Clermont-Ferrand, 24 avenue des Landais, 63177 Aubière, France
E-mail: morenas@clermont.in2p3.fr

Constantia Alexandrou, Tomasz Korzec, Giannis Koutsou
Department of Physics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus
E-mail: alexand@ucy.ac.cy, korzec@ucy.ac.cy, koutsou@ucy.ac.cy

Carsten Urbach
Institut für Elementarteilchenphysik, Fachbereich Physik, Humbolt Universität zu Berlin, D-12489, Berlin, Germany
E-mail: Carsten.Urbach@physik.hu-berlin.de

Dru Renner, Karl Jansen
DESY, Platanenallee 6, 15738 Zeuthen, Germany
E-mail: dru.renner@desy.de, Karl.Jansen@desy.de

We present results on the nucleon axial charge and the lowest moment $\langle x \rangle$ of the quark distribution using two degenerate flavors of light dynamical twisted mass fermions. The calculation is performed for pion masses in the range of 500 MeV to 300 MeV on a lattice of spatial size of about 2.1 fm.

The XXVI International Symposium on Lattice Field Theory
July 14 - 19, 2008
Williamsburg, Virginia, USA

[*] Speaker.
[†] For the ETM Collaboration

PoS(LATTICE 2008)162

1. Introduction

The twisted mass formulation of lattice QCD (tmQCD) provides a promising framework that allows for automatic $\mathscr{O}(a)$ improvement, infrared regularization of small eigenvalues and fast dynamical simulations [1]. Like in our previous work on the light baryon masses [2] we use the tree-level Symanzik improved gauge action and work at maximal twist to realize $\mathscr{O}(a)$-improvement.

The fermion action for two degenerate flavors of quarks in twisted mass QCD is given by

$$S_F = a^4 \sum_x \bar{\psi}(x) \left(D_W[U] + m_0 + i\mu\gamma_5\tau^3 \right) \psi(x) \tag{1.1}$$

with $D_W[U]$ the massless Dirac operator, m_0 the critical bare untwisted quark mass and μ the bare twisted mass. Together with the Wilson term, the twisted mass term in the fermion action of Eq. (1.1) breaks isospin symmetry since the u- and d-quarks differ by having opposite signs for the μ-term. This isospin breaking is a cutoff effect of $\mathscr{O}(a^2)$.

Nucleon form factors, parton distribution functions (PDF) and generalized parton distribution functions (GPD) are being and will continue to be measured in experimental programs at JLab and CERN [3, 4, 5]. Our long-term goal is to evaluate these observables within twisted mass QCD.

2. Lattice techniques

We use the standard interpolating field for the nucleon given by

$$J_p \;=\; \varepsilon_{abc} \left(u_a^T C \gamma_5 d_b \right) u_c$$

In the evaluation of three-point functions suppressing sufficiently excited state contributions is crucial in order to obtain accurate results. We apply Gaussian smearing to each quark field, $q(\mathbf{x},t)$:

$$q^{\text{smear}}(\mathbf{x},t) = \sum_{\mathbf{y}} F(\mathbf{x},\mathbf{y};U(t)) q(\mathbf{y},t) \tag{2.1}$$

using the gauge invariant smearing function

$$F(\mathbf{x},\mathbf{y};U(t)) = (1+\alpha H)^n (\mathbf{x},\mathbf{y};U(t)) \tag{2.2}$$

constructed from the hopping matrix,

$$H(\mathbf{x},\mathbf{y};U(t)) = \sum_{i=1}^{3} \left(U_i(\mathbf{x},t)\delta_{\mathbf{x},\mathbf{y}-i} + U_i^\dagger(\mathbf{x}-i,t)\delta_{\mathbf{x},\mathbf{y}+i} \right) \tag{2.3}$$

where the U fields are computed from the gauge fields via APE smearing. The parameters α and n entering the smearing function $F(\mathbf{x},\mathbf{y};U(t)$ are determined by requiring that the root mean square (r.m.s) radius obtained is in the range of 0.3-0.4 fm [2, 6].

The three-point functions that we need to evaluate are of the form

$$G_\mu(\Gamma^\nu,\vec{q},t) = \sum_{\vec{x},\vec{x}_f} e^{i\vec{x}\cdot\vec{q}}\, \Gamma^\nu_{\beta\alpha} \langle J_\alpha(t_f,\vec{x}_f) \mathscr{O}_\mu(t,\vec{x}) \bar{J}_\beta(0) \rangle \tag{2.4}$$

PoS(LATTICE 2008)162

$\beta = 3.9$					
$24^3 \times 48$, $L_s = 2.1$ fm	μ	0.0040	0.0064	0.0085	0.010
	m_π (GeV)	0.3131(16)	0.3903(9)	0.4470(12)	0.4839(12)
	no. of confs	419	184	346	173

Table 1: Parameters used in our calculation.

where for the evaluation of the axial charge we use [7]

$$\Gamma^\nu = \frac{1}{4}[1+\gamma_0]\gamma_5\gamma_\nu \;\; \nu = 1,2 \text{ or } 3 \tag{2.5}$$

For the lowest moment $\langle x \rangle$ we use

$$\Gamma^3 = \frac{1+\gamma_0}{2}\frac{1-i\gamma_5\gamma_3}{2} \tag{2.6}$$

The three-point functions of interest here are computed by evaluating the sequential propagator through the sink. This requires to fix the time separation between the source and sink (12 in our case) as well as the hadron state at the sink.

For the evaluation of the axial charge we take

$$\mathscr{O}_\mu = A_\mu(x) = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$$

For the lowest moment $\langle x \rangle$ we consider only the isovector combination since this has no contributions from the quark disconnected diagrams as one approaches the continuum limit. The twist-2 operator that we need to insert to evaluate the lowest moment of the quark distribution function $\langle x \rangle$ is given by

$$\mathscr{O}_{44}(x) = \frac{1}{2}\bar{u}(x)[\gamma_4 \overset{\leftrightarrow}{D}_4 - \frac{1}{3}\sum_{k=1}^{3}\gamma_k \overset{\leftrightarrow}{D}_k]u(x) \tag{2.7}$$

where $D_\mu = \frac{1}{2}(\bigtriangledown_\mu + \bigtriangledown_\mu^*)$ with $\bigtriangledown_\mu$ ($\bigtriangledown_\mu^*$) being the usual forward (backward) covariant derivative on the lattice. With the above operator, no external momentum is needed in our calculation, which is advantageous since an external momentum increases the noise to signal ratio.

3. Renormalization

The quantities g_A and $\langle x \rangle$ that we measure on the lattice need to be renormalized. This is done with the RI-MOM method, which is a non-perturbative, mass independent, renormalization scheme proposed in Ref. [10]. As the renormalization is defined by the quark vertex one needs to fix the gauge to get a non-zero result. In this work we use the Landau gauge. The quark propagators computed in our previous work are not recalculated but they are gauge transformed to Landau gauge once the gauge transformation is determined from the gauge fields. Because O_{44} is a non-local operator, one additional inversion is needed to get the renormalization constant $Z_{44}^{RI'}$. The hypercubic artifacts for this quantity have been reduced using the technique described in Ref. [8]. In Fig. 1 we compare this method with the “democratic method” (“cylindrical” cut on the values of p, keeping only those that are within a prescribed distance of the diagonal).

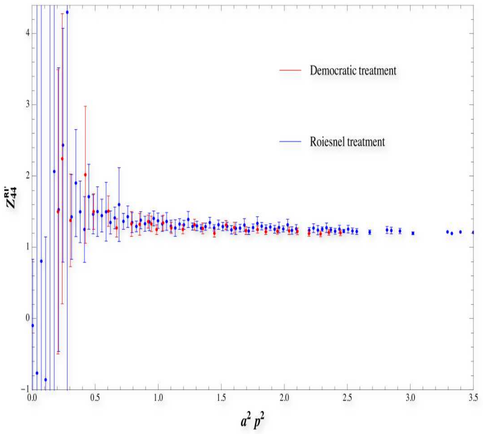

Figure 1: Comparison between democratic (red) and deSoto/Roiesnel (blue) treatment

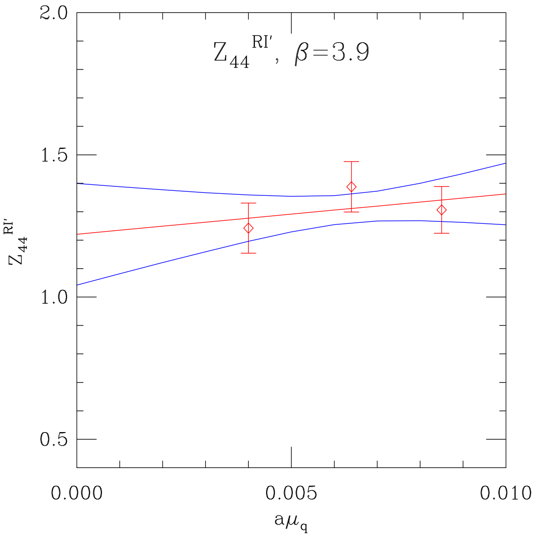

Figure 2: $Z_{44}^{RI'}$ for different quark masses.

The renormalization constant in the RI' and $\overline{MS}$ schemes are related by (we omit the index $_{44}$ for simplicity)

$$Z^{\overline{MS}} = Z^{RI'} Z_{RI'}^{\overline{MS}}$$

where $Z_{RI'}^{\overline{MS}}$ is taken from Eq. (5) of Ref. [11]. Using Λ_{QCD} = 250 MeV and setting the scale at μ=2 GeV we get $Z_{RI'}^{\overline{MS}}$ (2 GeV) = 0.909. Our lattice results given in Fig. 2 are extrapolated to the chiral limit. We estimate a value of $Z^{RI'}$ (2 GeV) = 1.22(18) and therefore $Z^{\overline{MS}}$ (2 GeV) = 1.11(16).

The same renormalization scheme is used to compute the axial renormalization constant in Ref. [9]. The value they obtain is $Z_A = 0.76(1)$, which we use here.

4. Results

In Fig. 3 we show our result for the renormalized value of g_A together with results from the LHPC [12] and RBC+UKQCD [13] collaborations. Statistical errors are estimated using the jackknife method. Our results are compatible with the results from these collaborations. It is important to note that our values span the light quark region. Our goal is to obtain results for a lighter quark mass bridging further the gap to the physical point. A chiral extrapolation will then be performed. An additional merit of our computation is the fact that we use a finer lattice than the other collaborations ensuring small cutoff effects. Our lattice spacing determined from f_π is 0.0855(6) fm [14] in agreement with the value of 0.0889(12)(14) extracted from the nucleon mass [2].

In Fig. 4, we show lattice results for $\langle x \rangle_{u-d}$ without multiplying by the $Z_{44}^{RI'}$-factor. Assuming tentatively a fit by a constant we get at the physical point $\langle x \rangle_{u-d}$ = 0.34(6). If we take into account the renormalization factor, we obtain $\langle x \rangle_{u-d}^{\overline{MS}}$(2GeV) = 0.37(8). A recent result [15] gives $\langle x \rangle_{u-d}^{\overline{MS}}$(2GeV) = 0.282(19). It is well known that with present pion masses lattice simulations provide values significantly larger than experiment [16, 17].

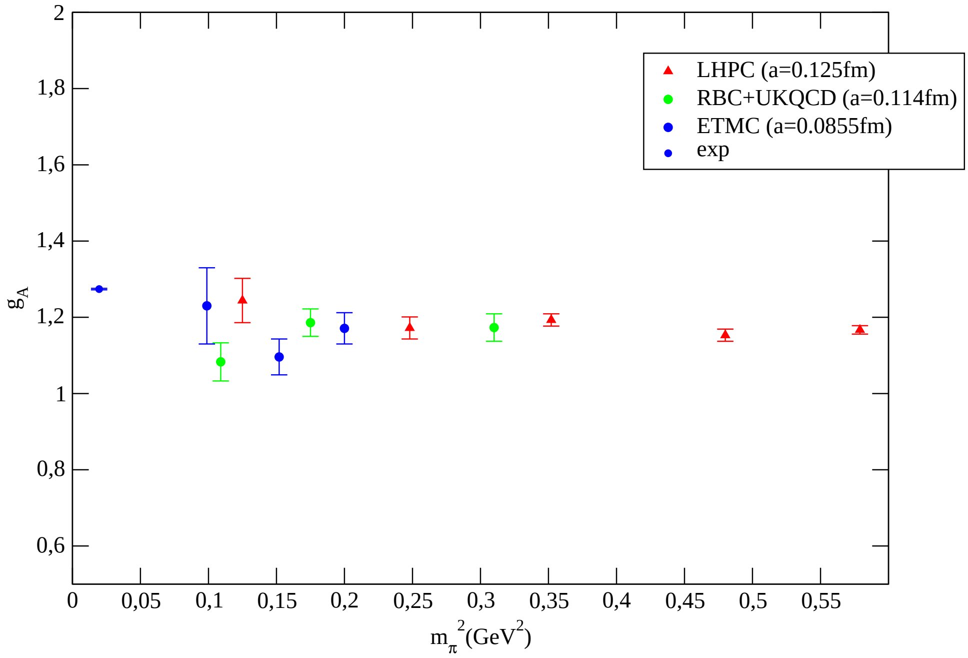

Figure 3: Nucleon axial charge as a function of the pion mass squared

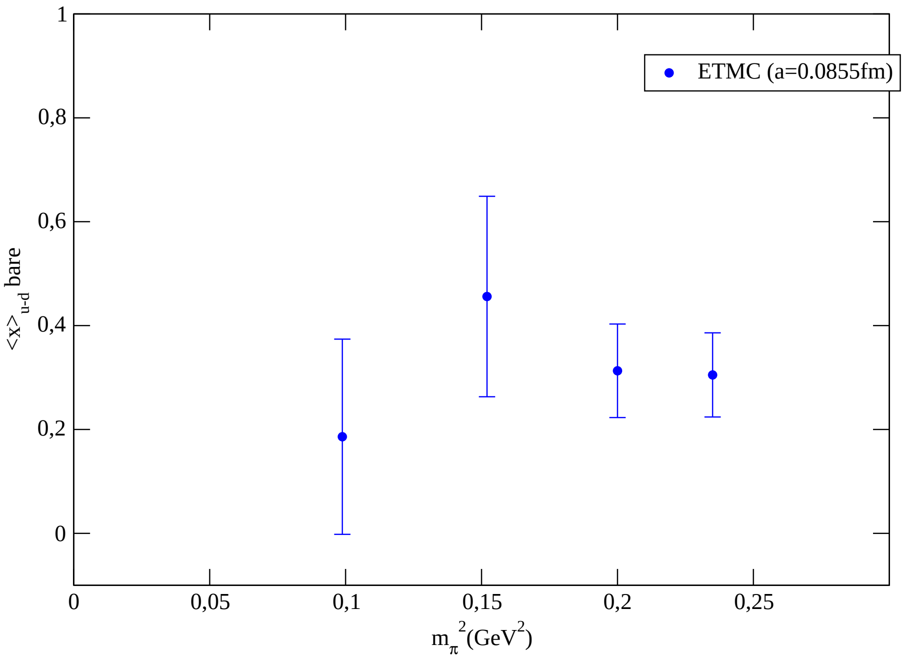

Figure 4: The lowest moment $\langle x \rangle_{u-d}$ for the proton as a function of the pion mass squared.

5. Summary

We have shown first results on the nucleon axial charge and lowest moment $< x >$ using two flavors of dynamical twisted mass fermions. Results are obtained for pion masses as low as about 300 MeV. A chiral extrapolation of these results to the physical point will be carried out once the statistical errors are reduced and a result at a lighter quark mass is obtained. Since our simulations are done on a fine lattice we expect cutoff effects to be small thus allowing us to use continuum chiral perturbation theory.

Acknowledgments

The calculations are performed at CC-IN2P3 and on the computers of the CCRT (CEA) computing center. We thank them for their support.

References

[1] R. Frezzotti and G.C.Rossi, JHEP 0408 (2004) 007, K. Jansen *et al.* Phys. Lett. **B619** (2005) 184; F. Farchioni *et al.* Eur. Phys. J. C39 (2005) 421; F. Farchioni *et al.*, Eur. Phys. J. C47 (2006) 453;

[2] C. Alexandrou *et al.* [European Twisted Mass Collaboration], Phys. Rev. D **78**, 014509 (2008) [arXiv:0803.3190 [hep-lat]].

[3] S. Chen *et al.* [CLAS Collaboration], Phys. Rev. Lett. **97** (2006) 072002 [arXiv:hep-ex/0605012].

[4] I. A. Qattan *et al.*, Phys. Rev. Lett. **94** (2005) 142301 [arXiv:nucl-ex/0410010].

[5] E. Burtin, N. d'Hose, P. A. M. Guichon and J. Marroncle, Nucl. Phys. A **721** (2003) 368.

[6] C. Alexandrou *et al.* [ETM Collaboration], PoS **LAT2007**, 087 (2007) [arXiv:0710.1173 [hep-lat]].

[7] C. Alexandrou *et al.*, PoS(LATTICE2008) 139 (2008).

[8] F. de Soto and C. Roiesnel, JHEP **0709** (2007) 007 [arXiv:0705.3523 [hep-lat]].

[9] P. Dimopoulos, R. Frezzotti, G. Herdoiza, A. Vladikas, V. Lubicz, S. Simula and M. Papinutto, PoS **LAT2007** (2007) 241 [arXiv:0710.0975 [hep-lat]].

[10] G. Martinelli, C. Pittori, C. T. Sachrajda, M. Testa and A. Vladikas, Nucl. Phys. B **445** (1995) 81 [arXiv:hep-lat/9411010].

[11] M. Gockeler *et al.*, Nucl. Phys. Proc. Suppl. **73** (1999) 291 [arXiv:hep-lat/9809001].

[12] R. G. Edwards *et al.* [LHPC Collaboration], Phys. Rev. Lett. **96** (2006) 052001 [arXiv:hep-lat/0510062].

[13] T. Yamazaki *et al.* [RBC+UKQCD Collaboration], Phys. Rev. Lett. **100** (2008) 171602 [arXiv:0801.4016 [hep-lat]].

[14] C. Urbach, PoS(LATTICE2007) 022 (2007).

[15] H. W. Lin, T. Blum, S. Ohta, S. Sasaki and T. Yamazaki, arXiv:0802.0863 [hep-lat].

[16] D. Brommel *et al.*, arXiv:0804.4706 [hep-lat].

[17] M. Gockeler *et al.*, Eur. Phys. J. A **32** (2007) 445 [arXiv:hep-lat/0609001].