PROCEEDINGS OF SCIENCE

Theory overview

Alexander Lenz*
IPPP, Durham University, United Kingdom
E-mail: alexander.lenz@durham.ac.uk

We set the scene for theoretical issues in charm physics that were discussed at CHARM 2016 in Bologna. In particular we emphasize the importance of improving our understanding of standard model contributions to numerous charm observables and we discuss also possible tests of our theory tools, like the Heavy Quark Expansion via the lifetime ratios of *D*-mesons.

VIII International Workshop On Charm Physics
5-9 September, 2016
Bologna, Italy

*Speaker.

PoS(CHARM2016)003

1. Introduction

First of all I would like to thank the organisers for giving me the possibility of opening this conference and for choosing such an inspiring venue, as the Convent of San Domenico. Besides having a long intellectual tradition, going back to scholars like Albertus Magnus, Meister Eckart and Thomas Aquinas, these sages had an interesting approach of dealing with different-minded people, which we might re-consider nowadays, in particular when dealing with politicians, who endanger the scientific progress with nationalistic and racist attitudes.
The idea of this talk was not to anticipate any of the results that will be shown later in the conference; they will be covered by the individual talks and by the theory summary [1]. Moreover, to give a further indication of the broad range of theoretical activities in this field, I mostly quote results from groups that are not present at CHARM 2016. Marco will present the corresponding experimental introduction [2], including historical remarks. The topics discussed in Bologna were grouped into the following sessions: *Heavy Ions*, *Multi-body hadronic decays and amplitude analysis*, *Leptonic, semi-leptonic and rare decays (CKM elements)*, *Charm Baryon decays*, *Charmonium and Exotics, production and spectroscopy*, *CP violation, Mixing and non-leptonic decays*, *Open Charm production and spectroscopy* and *Future Prospects*. In order to give a systematic, brief overview of our field and its aims as well as to point out some directions, that might be important for the future development of our field I have chosen to re-categorise these topics into:

1. What is special about Charm?
   - the mass of the charm quark is neither heavy nor light; thus it is questionable if theory tools like the Heavy Quark Expansion ([3] or [4] for a review) or factorisation [5] work in the charm system.
   - the charm system is subjected to severe GIM [6] cancellations. This standard model peculiarity might not be present in extensions of it. Moreover such a strong numerical effect might also overshadow tests of the applicability of theory tools.
   - due to different couplings and parameters the investigation of charmed hadrons is complementary to the well-studied Kaons and B-mesons.
   - finally a very pragmatic motivation for studying the charm system: we have a huge amount of charm data, e.g. from LHCb or BESIII (see e.g. [7]) and there is much more to come from Belle II [8], the LHC-upgrade [9], BESIII [10] and PANDA [11].
2. Understanding of QCD: having a quantitative understanding of hadronic effects in the charm sector is absolutely crucial for any conclusions about possible new physics effects in the charm system; a very instructive example for that statement is the ΔA_{CP}-saga, see e.g. [12].
   - Spectroscopy, exotics: the theoretical description and understanding of bound-states including charm quarks is a very active research field, in particular since exotic states like penta-quarks have been established experimentally [13, 14]; we had nine dedicated theory talks in Bologna: [15], [16], [17], [18], [19], [20], [21], [22], [23].
   - Charm contributions in heavy ion physics might shed more light into the nature of the quark-gluon plasma and thus on cosmology, see e.g. [24], [25], [26], [27], [28] at this conference.

- Charm production is described by perturbative QCD, we had two presentations at CHARM 2016: [29], [30].
- Leptonic and semi-leptonic decays have the simplest possible hadronic structure, they depend on non-perturbative decay constants and form factors, which can be determined on the lattice or with sum rules. The lattice progress was described in [31]. These decays seem to be good candidates for new physics searches [32], [33].
- Hadronic decays are considerably more difficult to be described in theory, thus it is not clear whether tools like QCD factorisation give us any insight and one has to use assumptions like $SU(3)_F$-symmetry to make predictions. This topic was also intensively discussed at CHARM 2016: [39], [40], [41], [42], [43].
- Mixing: a naive application of the theory tools that work well in the B-system to the D-system gives results that are orders of magnitudes away from the experimental result. Here progress is urgently needed to make use of the relatively precise data - mixing was discussed in Bologna in [44], [45]. Lattice might turn out to yield promising results for D-mixing on a longer time scale [46]; on a shorter time scale a precise theoretical investigation of charm lifetimes [47], which is doable with current lattice technology, could shed light into the convergence properties of the HQE in the charm-system.

3. Determination of Standard Model parameters:

   - The CKM elements V_{cs} and V_{cd} are among the least well known mixing parameters; their measurement provides a test of the unitarity of the CKM matrix. The impact of charm physics to the CKM fit was discussed in [48].
   - The precise value of the charm quark mass m_c is needed for e.g. precision predictions in the b-quark sector. This topic was not discussed in Bologna.

4. Search for new physics effects in the charm sector are complementary to many of the current indirect search strategies:

   - D-meson decays (leptonic, semi-leptonic and hadronic ones) were discussed in that respect by [32, 33]. If new physics particles are heavy then our theory tools could work again well for the new contributions; unfortunately it is still very problematic to estimate the size of the Standard Model part.
   - A study of the Higgs-Yukawa coupling ($H \to c\bar{c}$) was suggested several times in the recent literature, see e.g. [34, 35, 36, 37, 38].
   - There are almost no studies of dark matter candidates that couple to the up-type quark sector, see [49].
   - Indirect charm contributions to quantities that are very sensitive to new physics effects are currently studied on the lattice, e.g. $g-2$ [50], ε_K [51, 52].

5. Our understanding of Quantum Mechanics might be improved by quantum coherent charm measurements; this was discussed in [53].

Due to a limitation in space we will not discuss all these topics in the proceedings.

2. What is special about Charm?

The masses of the charm and the bottom quarks are now very well determined. In [54] values of

$$\bar{m}_c(\bar{m}_c) = 1.267(11)\,\text{GeV}\,, \tag{2.1}$$
$$\bar{m}_b(\bar{m}_b) = 4.183(83)\,\text{GeV} \tag{2.2}$$

were obtained, using lattice QCD. The large value of the bottom quark mass enables an expansion of inclusive decay rates in the inverse of this value [3]:

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \ldots\,, \tag{2.3}$$

where Λ is a hadronic scale. The convergence of the HQE in the bottom sector was proven [55] by the agreement of experiment [56] and theory [57] (based on [58, 59, 60, 61, 62, 63, 64]) for the decay rate difference $\Delta\Gamma_s$ in the neutral B_s^0-system.

$$\Delta\Gamma_s^{\text{HFAG}} = (0.083 \pm 0.006)\text{ps}^{-1}\,, \qquad \Delta\Gamma_s^{\text{SM}} = (0.088 \pm 0.020)\text{ps}^{-1}\,. \tag{2.4}$$

The charm quark mass is roughly a factor of three smaller than the bottom-quark mass and thus much closer to the hadronic scale Λ_{QCD}. Hence it is questionable if the HQE is still converging, even if it does not seem unreasonable a priori. The experimental values for the mixing observables in the charm sector read [56]:

$$x_D^{\text{Exp.}} = \frac{\Delta M_D}{\Gamma_D} = (0.37 \pm 0.16)\cdot 10^{-2}\,, \qquad y_D^{\text{Exp.}} = \frac{\Delta\Gamma_D}{2\Gamma_D} = (0.66^{+0.07}_{-0.10})\cdot 10^{-2}\,. \tag{2.5}$$

To test the applicability of the HQE we simply adopt the formulae from the B-sector to the D-mesons [65] (including $\alpha_s(m_c)$- and Λ/m_c-corrections)

$$y_D^{\text{HQE}} \leq |\Gamma_{12}^D|\tau_D\,, \tag{2.6}$$
$$\Gamma_{12}^D = -\left(\lambda_s^2\Gamma_{12}^{ss} + 2\lambda_s\lambda_d\Gamma_{12}^{ds} + \lambda_d^2\Gamma_{12}^{dd}\right)\,. \tag{2.7}$$

λ_q denotes CKM structures and the Γ_{12}^{pq} are the loop contributions with an internal p- and q-quark.

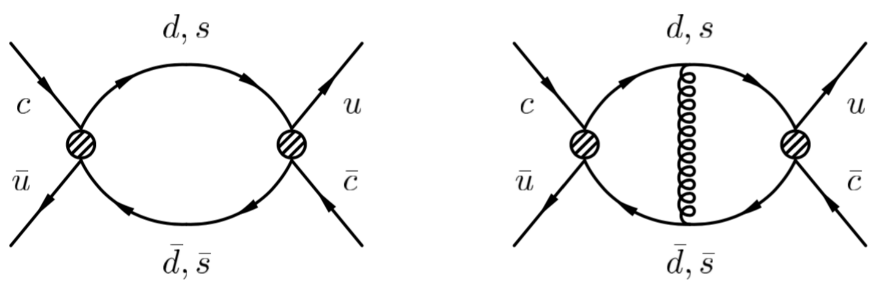


Figure 1: Contributions to Γ_{12} from operators of dimension 6 ($D = 6$). The leading order QCD diagram is shown in the left panel, an example for α_s corrections is shown in the right panel.

Considering only the s-quark contribution, we get

$$y_D^{\text{HQE}} \supset -\lambda_s^2\Gamma_{12}^{ss}\tau_D \approx 5.6\,y_D^{\text{Exp.}}\,. \tag{2.8}$$

Thus a single diagram gives a contribution that is larger than the experimental value. Considering now all three contributions and using in addition the unitarity of the CKM matrix, we find a severe GIM cancellation

$$y_D^{\rm HQE} \approx -\lambda_s^2 \left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd} \right) \tau_D \approx 1.7 \cdot 10^{-4} y_D^{\rm Exp.} , \tag{2.9}$$

pushing the HQE prediction far below the experimental value. Similar GIM cancellations appear also in rare penguin induced D-decays. Below we will discuss implications of this severe cancellations.

3. Understanding of QCD

For spectroscopy, exotics, heavy ions and charm production we refer the reader to the individual contributions [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30] and we concentrate here on meson decays and mixing. Leptonic decays, like $D_s^+ \to \mu^+ + \nu_\mu$, posses the simplest hadronic structure, which is parameterised by a decay constant f_{D_s}:

$$\langle 0 | \bar{c} \gamma_\mu \gamma_5 q | D_q(p) \rangle = i f_{D_q} p_{D_q}^\mu . \tag{3.1}$$

The theoretical determination with sum rules and lattice QCD of decay constants is quite advanced, see e.g. [66], and it agrees well with experimental measurements:

Model	$f_{D_s^+}$(MeV)	f_{D^+}(MeV)	$f_{D_s^+}/f_{D^+}$
Experiment (our averages)	257.5 ± 4.6	204.6 ± 5.0	1.258 ± 0.038
Lattice (HPQCD) [22]	$246.0 \pm 0.7 \pm 3.5$	$208.3 \pm 1.0 \pm 3.3$	$1.187 \pm 0.004 \pm 0.012$
Lattice (FNAL+MILC) [23]	$246.4 \pm 0.5 \pm 3.6$	$209.2 \pm 3.0 \pm 3.6$	1.175 ± 0.019
PQL [24]	244 ± 8	197 ± 9	1.24 ± 0.03
QCD sum rules [25]	205 ± 22	177 ± 21	$1.16 \pm 0.01 \pm 0.03$
QCD sum rules [26]	$245.3 \pm 15.7 \pm 4.5$	$206.2 \pm 7.3 \pm 5.1$	$1.193 \pm 0.025 \pm 0.007$
QCD sum rules [27]	246 ± 6	204 ± 6	1.21 ± 0.04
QCD sum rules [28](I)	241 ± 12	208 ± 11	1.16 ± 0.07
QCD sum rules [28](II)	258 ± 13	211 ± 14	1.22 ± 0.08
QCD sum rules [29]	238^{+13}_{-23}	201^{+12}_{-13}	$1.15^{+0.04}_{-0.05}$
Field correlators [30]	260 ± 10	210 ± 10	1.24 ± 0.03
Light front [31]	268.3 ± 19.1	206 (fixed)	1.30 ± 0.04

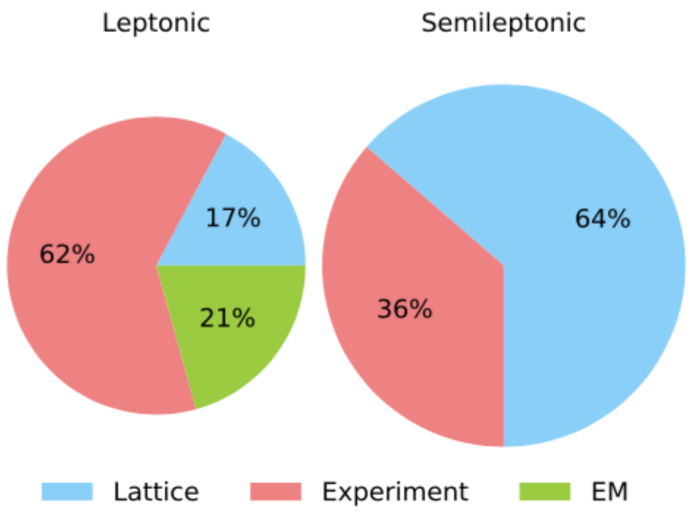
Table from Ref. [66].

From a hadronic point of view the next complicated class of decays are semi-leptonic decays. Here the non-perturbative part is parameterised by form factors

$$\langle K | V^\mu | D \rangle = f_+(q^2) \left(p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_D^2 - M_K^2}{q^2} q^\mu , \tag{3.2}$$

where f_+ and f_0 denote the form factors, that depend on the momentum transfer. To get an idea of the currently achieved theoretical precision for decay constants and form factors we show a plot from a talk of Steven Gottlieb at LATTICE 2016 [67].

PoS(CHARM2016)003



Comparison of contributions to $|V_{cs}|$ errors from the leading leptonic decay[1] and semileptonic decay[2] determinations. Radius is proportional to total error.

Even more complicated are purely hadronic decays like $D \to \pi\pi$ or $D \to KK$. It is not clear at all, that matrix elements like

$$\langle \pi\pi | Q | D \rangle , \tag{3.3}$$

where Q is a four-quark operator, factorise. To make nevertheless theoretical statements, assumptions like $SU(3)_F$-symmetry are regularly used in the literature, see e.g. [68, 69, 70, 71, 72, 73, 74] for some recent (2013 onward) references. In the long term future this problem might be solvable with lattice QCD, by an extension [75] of methods that have shown to be very successful for hadronic Kaon decays [51].

Finally we are coming back to mixing. The off-diagonal elements of the matrix describing the mixing of neutral D-mesons can be expressed as

$$2M_D \left(M_{12}^D - \frac{i}{2}\Gamma_{12}^D \right) = \langle D^0 | \mathcal{H}_{\rm eff}^{|\Delta C|=2} | \bar{D}^0 \rangle + \sum_n \frac{\langle D^0 | \mathcal{H}_{\rm eff}^{|\Delta C|=1} | n \rangle \langle n | \mathcal{H}_{\rm eff}^{|\Delta C|=1} | \bar{D}^0 \rangle}{M_D - E_n + i\varepsilon} . \tag{3.4}$$

The first term on the r.h.s. is short distance dominated. The necessary matrix elements of the 4-quark operators have been determined on the lattice [76, 77]; this part is thus relatively well understood. It is also interesting to note that heavy new physics particles contribute predominantly to the short distance part. The second term on the r.h.s. is dominated by light internal particles. In the B-system the corresponding contribution to M_{12} turned out to be negligible (due to the CKM structure and the large value of the top-quark mass) and M_{12} is given to a very good approximation by the $|\Delta B = 2|$ contribution alone. This is not the case anymore in the D-system. On the other hand, Γ_{12} is governed by on-shell intermediate particles, hence only the $|\Delta B, C = 1|$-parts are contributing, In the case of B-mixing it turned out to be possible to use quark hadron duality and perform successfully an expansion in Λ/m_b, the Heavy Quark Expansion. In the D-system one expects the expansion parameter to be a factor three larger, which might still be ok. A related way of looking at that, is to compare the remaining phase space in the decay channels contributing to the decay rate difference with the hadronic scale.

$$M_{B_s^0} - 2M_{D_s^{(*)}} = 1.43(1.15)\,\mathrm{GeV}\,, \tag{3.5}$$

$$M_{B_s^0} - M_{J/\psi} - M_\phi = 1.25\,\mathrm{GeV}\,, \tag{3.6}$$

$$M_D - 2M_K = 0.88\,\mathrm{GeV}\,, \tag{3.7}$$

$$M_D - 2M_\pi = 1.59\,\mathrm{GeV}\,. \tag{3.8}$$

These numbers for $\Delta\Gamma_s$ and $\Delta\Gamma_D$ are quite similar, so a priori it seems not obvious that the HQE does not converge in the charm system, on the other we have seen above that the leading terms of the HQE give results that are about four orders of magnitude below the experimental value. Instead of the inclusive approach which assumes quark hadron duality one can try to use the exclusive approach, where the sum over all possible final states into which both the D^0-meson and the $\bar{D}^0$-meson can decay, has to be performed. This is obviously even more challenging than calculating only one hadronic D-meson decay. Nevertheless this approach together with several simplifying assumptions was used in [78, 79] to predict roughly the experimental values of the charm mixing observables. Here the next step would be to reduce the number of assumptions and make the theory predictions more realistic; there are indications that on a long time scale this issue could be solved on the lattice by further developments of the methods from Hansen and Sharpe [75].

But even, if it turns out in future that the exclusive approach will reproduce the experimental D-mixing values, it is not clear, why the inclusive approach is failing by about four orders of magnitude, despite working so well in the B sector and despite having an expansion parameter that is only a factor of about three larger. Above we already observed that the individual diagrams for Γ_{12}^D give values larger than y^{Exp} and only the combination of all three contributing diagrams is more or less vanishing due to GIM cancellations. This old observation was revived recently in [80], where it was found that a modification of individual diagrams of the order of 20% due to some unknown duality violating effect, would be sufficient to lift the GIM suppression so much, that the experimental value of y could be reproduced by the leading term in the HQE.

In the same spirit, it was already argued several years ago [81, 82, 83] that the GIM suppression might be lifted in higher orders of the HQE. If the lifting of the GIM suppression is more pronounced than the suppression due to Λ/m_c, then the dominant contribution of the HQE prediction for Γ_{12}^D might actually stem from dimension 9 or dimension 12 contributions, see also [65]. A first step in that direction was already done in [84], which seems to indicate that dimension nine is in fact larger than the leading dimension six contribution, but unfortunately still considerably below experiment.

Next one could continue to determine the dimension twelve contributions and thus test the above idea of the dominance of higher orders in the HQE. A severe limitation of this approach will, however, be the treatment of the unknown matrix elements of the 8-fermion operators. Except naive factorisation there is currently no adequate theory tool in sight. An alternative test of the convergence of the HQE in the charm-system would be the investigation of observables that are free of GIM-cancellations. Here lifetime ratios are the prime candidates. The experimental value of the lifetime ratio shows quite a large deviation from one

$$\frac{\tau(D^+)}{\tau(D^0)}^{\mathrm{Exp}} = 2.536 \pm 0.019\,. \tag{3.9}$$

This number does, however, not necessarily point towards a 150% correction in the HQE, a 40% correction could also easily do the job, via $(1+0.4)/(1-0.4) = 2.3$. The perturbative part of the HQE prediction for this lifetime ratio is known up to next-to-leading order in QCD and in the parameter Λ/m_c [47]; unfortunately the non-perturbative matrix elements of the arising 4-quark operators

$$Q^q = \bar{c}\gamma_\mu(1-\gamma_5)q \cdot \bar{c}\gamma^\mu(1-\gamma_5)q\,, \tag{3.10}$$

$$Q_S^q = \bar{c}(1-\gamma_5)q \cdot \bar{c}(1+\gamma_5)q\,, \tag{3.11}$$

$$T^q = \bar{c}\gamma_\mu(1-\gamma_5)T^a q \cdot \bar{c}\gamma^\mu(1-\gamma_5)T^a q\,, \tag{3.12}$$

$$T_S^q = \bar{c}(1-\gamma_5)T^a q \cdot \bar{c}(1+\gamma_5)T^a q \tag{3.13}$$

have not yet been determined. This task seems to be perfectly doable with current lattice technology. Making some simplifying assumptions for the unknown matrix elements Ref.[47] obtained

$$\frac{\tau(D^+)}{\tau(D^0)}^{\rm HQE} = 2.2 \pm 1.7^{\rm hadronic}{}^{+0.3\,\rm scale}_{-0.7} \pm 0.1^{\rm parametric}\,. \tag{3.14}$$

This result is promising, but unfortunately not conclusive, due to the huge uncertainties related to the unknown matrix elements. Here a lattice study could shed very valuable insight into the applicability of the HQE. Finally it is entertaining to note that there is still the possibility of having found new physics in D-mixing without having noticed it yet.

4. Determination of Standard model parameters:

The direct determination of the CKM elements V_{cs} and V_{cd} still suffers from considerable uncertainties. PDG [85] quotes values of

$$V_{cd} = 0.225 \pm 0.008\,, \tag{4.1}$$

$$V_{cs} = 0.986 \pm 0.016\,, \tag{4.2}$$

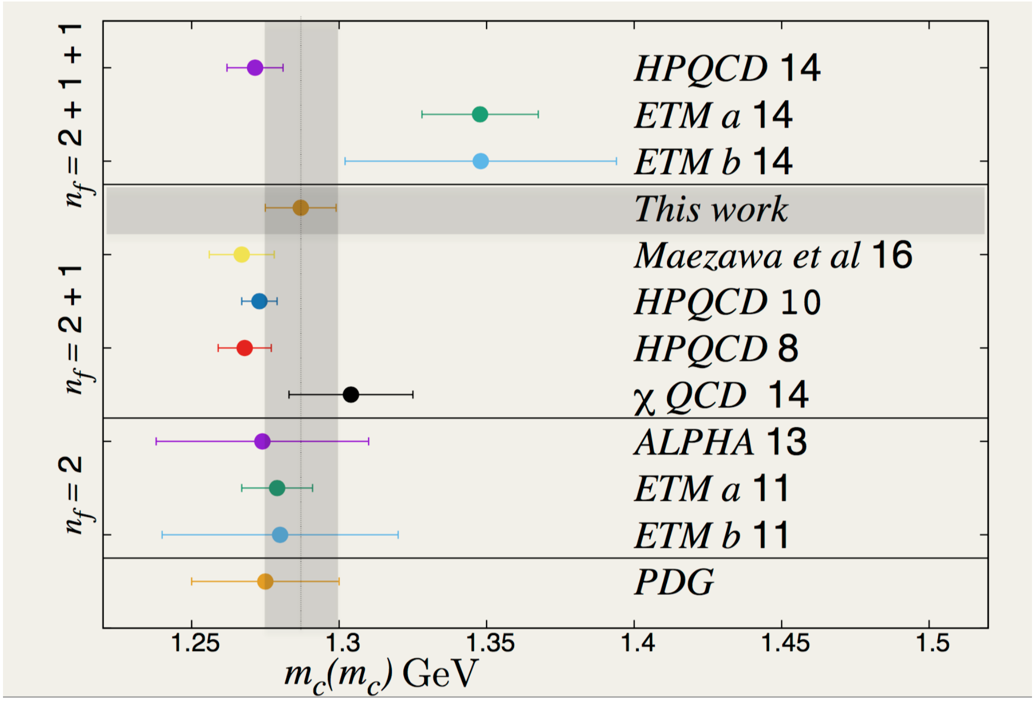
$$V_{cb} = 0.0411 \pm 0.0013\,. \tag{4.3}$$

This leads to the following test of the unitarity of the CKM matrix

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.024 \pm 0.032 = 1 + (0.15)^2 \pm (0.18)^2\,. \tag{4.4}$$

Thus there is still space for sizable new physics effects and the corresponding CKM values have to be determined more precise in future. The status quo of CKM fits will be reviewed by Derkach [48].

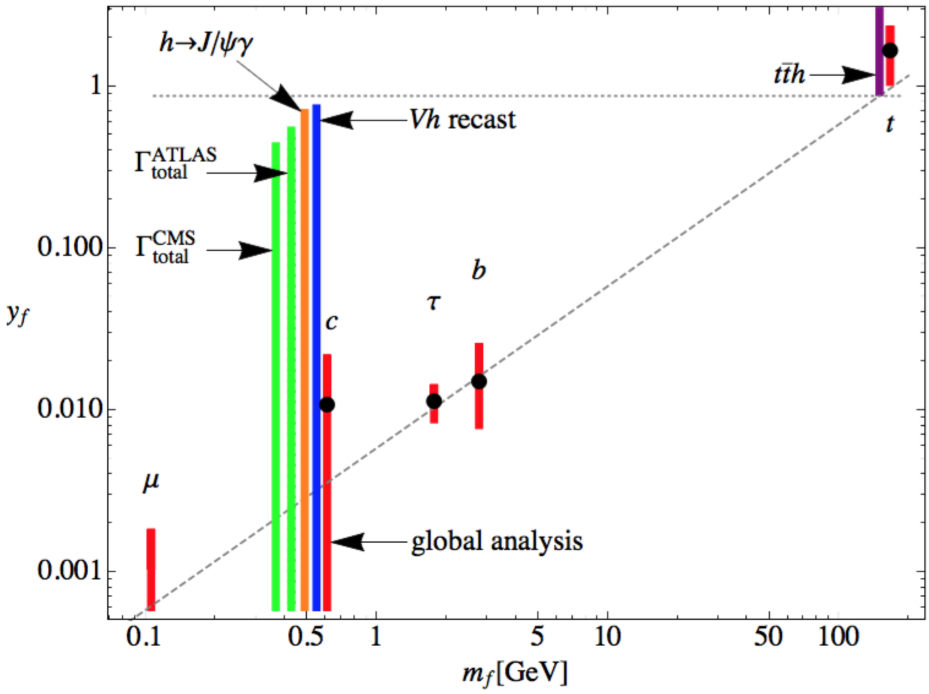
The charm quark mass m_c has been determined by many groups and has obtained an impressive precision, which is necessary for making precise predictions in other fields, like b-physics, where the charm quark mass is an important input parameter. An overview of lattice determinations was shown at LATTICE 2016 by Nakayama [86]:



This topic was not discussed in Bologna.

5. Search for new physics:

D-meson decays and *D*-mixing are places where new physics effects could lead to significant contributions. If the new degrees of freedom are heavy, i.e. of the order of the weak scale or larger, then we could use effective theories to integrate the new particles out and our theory tools would be in a similar good shape as in the *B*-system. The larger value of the strong coupling at the scale of charm quark mass will, however, still be a drawback, as well as the uncertainty related to the detailed size of the standard model contribution. On the other hand the new contributions could not be affected by the severe GIM cancellations present in the Standard Model part of *D*-mixing and some rare D-decays, leading thus to interesting bounds on the new couplings. New physics effects to *D*-meson decays were discussed in [32, 33]. There is also some ongoing interest in finding bounds on the Yukawa-couplings of the charm quark [34, 35, 36, 37, 38]. The current bounds on the Higgs couplings were e.g. presented by Joachim Brod at BEAUTY 2016



Here the charm coupling is severely unconstrained and leaves plenty of room for new effects. Indirect charm contributions are also crucial for precise Standard Model values of quantities that are very sensitive to new effects and there we currently find deviations of the order of 3 standard deviations, e.g. the anomalous magnetic moment of the muon $g-2$ and indirect CP-violation in Kaon-mixing, denoted by ε_K, see e.g. [51, 52].

6. Conclusion

Further indications for the fact that charm phenomenology is a very rich field can be found in all the contributions to CHARM 2016, including the experimental summary [87].
We concentrated in these proceedings on the applicability of our theory tools to the charm sector. The seemingly obvious failure of the HQE for charm mixing, might also have different sources than a simple inapplicability:

- Quark hadron duality violating effects as low as 20 % could be the source of the discrepancy. Thus for other observables the HQE might still give decent estimates.
- Higher order terms in the HQE might be the dominant contribution due to a lifting of the GIM cancellations. Here again the HQE might still give good estimates for quantities that are not affected by severe GIM cancellations.
- Finally it is amusing to note, that it is still not completely excluded that we already observed new physics effects in D mixing.

The question whether the HQE gives reasonable estimates for charm observables that are not affected by strong GIM cancellations, can be well tested by theoretical studies of the D-meson lifetimes. The only missing theoretical ingredients for this test are matrix elements of 4-quark operators, that can be determined with current lattice technology. By the time of CHARM 2018 in Novosibirsk [88] we will hopefully be closer to an answer of these questions.

References

[1] Svjetlana Fajfer, these proceedings; talk at CHARM 2016.

[2] Marco Gersabeck, these proceedings; talk at CHARM 2016.

[3] V. A. Khoze, M. A. Shifman, N. G. Uraltsev and M. B. Voloshin, Sov. J. Nucl. Phys. **46** (1987) 112 [Yad. Fiz. **46** (1987) 181].
M. A. Shifman and M. B. Voloshin, Sov. Phys. JETP **64** (1986) 698 [Zh. Eksp. Teor. Fiz. **91** (1986) 1180].
I. I. Y. Bigi and N. G. Uraltsev, Phys. Lett. B **280** (1992) 271. doi:10.1016/0370-2693(92)90066-D
I. I. Y. Bigi, N. G. Uraltsev and A. I. Vainshtein, Phys. Lett. B **293** (1992) 430 Erratum: [Phys. Lett. B **297** (1992) 477] doi:10.1016/0370-2693(92)90908-M, 10.1016/0370-2693(92)91287-J [hep-ph/9207214].
B. Blok and M. A. Shifman, Nucl. Phys. B **399** (1993) 441 doi:10.1016/0550-3213(93)90504-I [hep-ph/9207236].
B. Blok and M. A. Shifman, Nucl. Phys. B **399** (1993) 459 doi:10.1016/0550-3213(93)90505-J [hep-ph/9209289].
M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B **459** (1999) 631 doi:10.1016/S0370-2693(99)00684-X [hep-ph/9808385].

[4] A. Lenz, Int. J. Mod. Phys. A **30** (2015) no.10, 1543005. doi:10.1142/S0217751X15430058
A. Lenz, arXiv:1405.3601 [hep-ph].

[5] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. **83** (1999) 1914 doi:10.1103/PhysRevLett.83.1914 [hep-ph/9905312].
M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **591** (2000) 313 doi:10.1016/S0550-3213(00)00559-9 [hep-ph/0006124].
M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **606** (2001) 245 doi:10.1016/S0550-3213(01)00251-6 [hep-ph/0104110].
M. Beneke and M. Neubert, Nucl. Phys. B **675** (2003) 333 doi:10.1016/j.nuclphysb.2003.09.026 [hep-ph/0308039].

[6] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D **2** (1970) 1285. doi:10.1103/PhysRevD.2.1285

[7] M. Gersabeck, PoS FWNP (2015) 001 [arXiv:1503.00032 [hep-ex]].
M. Gersabeck, Mod. Phys. Lett. A **27** (2012) 1230026 doi:10.1142/S0217732312300261 [arXiv:1207.2195 [hep-ex]].
I. Garzia [BESIII Collaboration], J. Phys. Conf. Ser. **631** (2015) no.1, 012043. doi:10.1088/1742-6596/631/1/012043

[8] Alan Schwartz, these proceedings; talk at CHARM 2016.

[9] Chris Parkes, these proceedings; talk at CHARM 2016.

[10] Xiaoyan Shen, these proceedings; talk at CHARM 2016.

[11] Gianluigi Boca, these proceedings; talk at CHARM 2016.

[12] A. Lenz, arXiv:1311.6447 [hep-ph].

[13] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **117** (2016) no.8, 082002 doi:10.1103/PhysRevLett.117.082002 [arXiv:1604.05708 [hep-ex]].

[14] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **117** (2016) no.8, 082003 Addendum: [Phys. Rev. Lett. **117** (2016) no.10, 109902] doi:10.1103/PhysRevLett.117.082003, 10.1103/PhysRevLett.117.109902 [arXiv:1606.06999 [hep-ex]].

[15] Alfredo Valcarce, these proceedings; talk at CHARM 2016.

[16] Pedro Gonzalez, these proceedings; talk at CHARM 2016.

[17] Graham Moir, these proceedings; talk at CHARM 2016.

[18] Timothy Burns, these proceedings; talk at CHARM 2016.

[19] Francisco Fernandez, these proceedings; talk at CHARM 2016.

[20] Gavin Cheung, these proceedings; talk at CHARM 2016.

[21] Alessandro Pilloni, these proceedings; talk at CHARM 2016.

[22] Sinead Ryan, these proceedings; talk at CHARM 2016.

[23] Nora Brambillak, these proceedings; talk at CHARM 2016.

[24] Frank Geurts, these proceedings; talk at CHARM 2016.

[25] Francois Arleo, these proceedings; talk at CHARM 2016.

[26] Andrea Beraudo, these proceedings; talk at CHARM 2016.

[27] Antonio Vairo, these proceedings; talk at CHARM 2016.

[28] Martin Cleven, these proceedings; talk at CHARM 2016.

[29] Johann Haidenbauer, these proceedings; talk at CHARM 2016.

[30] Wei Wang, these proceedings; talk at CHARM 2016.

[31] Aida El-Khadra, these proceedings; talk at CHARM 2016.

[32] Nejc Kosnik, these proceedings; talk at CHARM 2016.

[33] Ayan Paul, these proceedings; talk at CHARM 2016.

[34] G. Perez, Y. Soreq, E. Stamou and K. Tobioka, Phys. Rev. D **92** (2015) no.3, 033016 doi:10.1103/PhysRevD.92.033016 [arXiv:1503.00290 [hep-ph]].

[35] F. Bishara, U. Haisch, P. F. Monni and E. Re, arXiv:1606.09253 [hep-ph].

[36] F. J. Botella, G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, arXiv:1602.08011 [hep-ph].

[37] I. Brivio, F. Goertz and G. Isidori, Phys. Rev. Lett. **115** (2015) no.21, 211801 doi:10.1103/PhysRevLett.115.211801 [arXiv:1507.02916 [hep-ph]].

[38] M. König and M. Neubert, JHEP **1508** (2015) 012 doi:10.1007/JHEP08(2015)012 [arXiv:1505.03870 [hep-ph]].

[39] Jonas Rademacker, these proceedings; talk at CHARM 2016.

[40] Benoit Loiseau, these proceedings; talk at CHARM 2016.

[41] Satoshi Nakamura, these proceedings; talk at CHARM 2016.

[42] Patricia Magalhaes, these proceedings; talk at CHARM 2016.

[43] Pietro Santorelli, these proceedings; talk at CHARM 2016.

[44] Guido Martinelli, these proceedings; talk at CHARM 2016.

[45] Marco Ciuchinik, these proceedings; talk at CHARM 2016.

[46] Lorenza Riggio, these proceedings; talk at CHARM 2016.

[47] A. Lenz and T. Rauh, Phys. Rev. D **88** (2013) 034004 doi:10.1103/PhysRevD.88.034004 [arXiv:1305.3588 [hep-ph]].

[48] Denis Derkach, these proceedings; talk at CHARM 2016.

[49] T.Jubb, M. Kirk and A. Lenz in preparation.

[50] Hartmut Wittig, talk at LATTICE 2016.

[51] Chris Sachrajda, talk at KAON 2016.

[52] Xu Feng, talk at KAON 2016.

[53] Roy Briere, these proceedings; talk at CHARM 2016.

[54] Y. Maezawa and P. Petreczky, Phys. Rev. D **94** (2016) no.3, 034507 doi:10.1103/PhysRevD.94.034507 [arXiv:1606.08798 [hep-lat]].

[55] A. Lenz, arXiv:1205.1444 [hep-ph].

[56] web-update of Y. Amhis *et al.* [Heavy Flavor Averaging Group (HFAG) Collaboration], arXiv:1412.7515 [hep-ex].

[57] M. Artuso, G. Borissov and A. Lenz, arXiv:1511.09466 [hep-ph].

[58] A. Lenz and U. Nierste, JHEP **0706** (2007) 072 doi:10.1088/1126-6708/2007/06/072 [hep-ph/0612167].

[59] M. Beneke, G. Buchalla, A. Lenz and U. Nierste, Phys. Lett. B **576** (2003) 173 doi:10.1016/j.physletb.2003.09.089 [hep-ph/0307344].

[60] M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, JHEP **0308** (2003) 031 doi:10.1088/1126-6708/2003/08/031 [hep-ph/0308029].

[61] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Nucl. Phys. B **639** (2002) 389 doi:10.1016/S0550-3213(02)00561-8 [hep-ph/0202106].

[62] A. S. Dighe, T. Hurth, C. S. Kim and T. Yoshikawa, Nucl. Phys. B **624** (2002) 377 doi:10.1016/S0550-3213(01)00655-1 [hep-ph/0109088].

[63] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B **459** (1999) 631 doi:10.1016/S0370-2693(99)00684-X [hep-ph/9808385].

[64] M. Beneke, G. Buchalla and I. Dunietz, Phys. Rev. D **54** (1996) 4419 Erratum: [Phys. Rev. D **83** (2011) 119902] doi:10.1103/PhysRevD.54.4419, 10.1103/PhysRevD.83.119902 [hep-ph/9605259].

[65] M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, JHEP **1003** (2010) 009 doi:10.1007/JHEP03(2010)009 [arXiv:1002.4794 [hep-ph]].

[66] J. L. Rosner, S. Stone and R. S. Van de Water, [arXiv:1509.02220 [hep-ph]].

[67] Steven Gottlieb, talk at LATTICE 2016.

[68] S. Müller, U. Nierste and S. Schacht, Phys. Rev. D **92** (2015) no.1, 014004 doi:10.1103/PhysRevD.92.014004 [arXiv:1503.06759 [hep-ph]].

[69] M. Gronau, Phys. Rev. D **91** (2015) no.7, 076007 doi:10.1103/PhysRevD.91.076007 [arXiv:1501.03272 [hep-ph]].

[70] D. N. Gao, Phys. Rev. D **91** (2015) no.1, 014019 doi:10.1103/PhysRevD.91.014019 [arXiv:1411.0768 [hep-ph]].

[71] M. Jung and S. Schacht, Phys. Rev. D **91** (2015) no.3, 034027 doi:10.1103/PhysRevD.91.034027 [arXiv:1410.8396 [hep-ph]].

[72] P. Santorelli, EPJ Web Conf. **80** (2014) 00051 doi:10.1051/epjconf/20148000051 [arXiv:1410.1719 [hep-ph]].

[73] G. Hiller, M. Jung and S. Schacht, PoS EPS **-HEP2013** (2013) 371 [arXiv:1311.3883 [hep-ph]].

[74] F. Buccella, M. Lusignoli, A. Pugliese and P. Santorelli, Phys. Rev. D **88** (2013) no.7, 074011 doi:10.1103/PhysRevD.88.074011 [arXiv:1305.7343 [hep-ph]].

[75] M. T. Hansen and S. R. Sharpe, Phys. Rev. D **86** (2012) 016007 doi:10.1103/PhysRevD.86.016007 [arXiv:1204.0826 [hep-lat]].

[76] N. Carrasco *et al.* [ETM Collaboration], Phys. Rev. D **92** (2015) no.3, 034516 doi:10.1103/PhysRevD.92.034516 [arXiv:1505.06639 [hep-lat]].

[77] Andreas Kronfeld, talk at LATTICE 2016.

[78] A. F. Falk, Y. Grossman, Z. Ligeti and A. A. Petrov, Phys. Rev. D **65** (2002) 054034 doi:10.1103/PhysRevD.65.054034 [hep-ph/0110317].

[79] A. F. Falk, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys. Rev. D **69** (2004) 114021 doi:10.1103/PhysRevD.69.114021 [hep-ph/0402204].

[80] T. Jubb, M. Kirk, A. Lenz and G. Tetlalmatzi-Xolocotzi, arXiv:1603.07770 [hep-ph].

[81] H. Georgi, Phys. Lett. B **297** (1992) 353 doi:10.1016/0370-2693(92)91274-D [hep-ph/9209291].

[82] T. Ohl, G. Ricciardi and E. H. Simmons, Nucl. Phys. B **403** (1993) 605 doi:10.1016/0550-3213(93)90364-U [hep-ph/9301212].

[83] I. I. Y. Bigi and N. G. Uraltsev, Nucl. Phys. B **592** (2001) 92 doi:10.1016/S0550-3213(00)00604-0 [hep-ph/0005089].

[84] M. Bobrowski, A. Lenz and T. Rauh, arXiv:1208.6438 [hep-ph].

[85] C. Patrignani, Chin. Phys. C **40** (2016) no.10, 100001. doi:10.1088/1674-1137/40/10/100001

[86] Katsumasa Nakayama, talk at LATTICE 2016.

[87] Angelo di Canto, these proceedings; talk at CHARM 2016.

[88] Alexander Bondar, these proceedings; talk at CHARM 2016.