PROCEEDINGS OF SCIENCE

Multi-Meson Model applied to $D^+ \to K^+K^-K^+$

Patricia C. Magalhães[*†]
CBPF - Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, RJ, Brazil
E-mail: pmagalhaes@cbpf.br

R. T. Aoude, A. C. dos Reis
CBPF - Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, RJ, Brazil

M. R. Robilotta
Instituto de Física, Universidade de São Paulo, São Paulo, SP, Brazil

The description of processes involving multi-body nonleptonic decays is very challenging. Data sets are very large nowadays, and demand better models. In this work we focus on three-kaon weak matrix elements $\langle (KKK)^+ | A_\mu | 0 \rangle$, expressed as a relatively simple structure, which generalizes the concept of form factor. This is particularly important for $D^+ \to K^+K^-K^+$ decay considering the dominance of annihilation weak topology. Here, we present the first version of a model to $D^+ \to K^+K^-K^+$ decay which encompass naturally all final states topologies, including a proper multi-particle structures that cannot be decomposed into simpler two-body processes. All the structures are derived from a chiral effective theory and dressed with coupled channels where appropriate. The final amplitude contain only 3 free parameters related to $f_0(980)$ that are predicted by the theory but needs to be fine-tuned by a fit to data. This approach represents a significant improvement when compared to isobar model, often employed in analyses of heavy-meson decay data.

VIII International Workshop On Charm Physics
5-9 September, 2016
Bologna, Italy

[*]Speaker.
[†]this work of was supported by CNPq Grant No. 157857/2015- 8.

PoS(CHARM2016)086

1. Introduction

Three-body nonleptonic decays of heavy-flavoured mesons are sequential processes, dominated by intermediate resonant states that requires a full amplitude analysis to be determined. The usual experimental approach is the isobar model. However, the analysis of the gigantic samples of B and D decays collected by the LHCb and Bes III experiments demands better models. A particularly important issue is the representation of the nonresonant component of the decay amplitude. In general, the nonresonant amplitude used to fit data is parametrized by ad hoc functions that are not compromised with any theory.

In a recent work[1], we propose a model for the decay $D^+ \to K^+K^-K^+$ as an alternative to isobar model, with few free parameter predicted by the theory to be fine-tuned by a fit to data. The advantage of this specific process is the dominance of annihilation weak topology where the process can be understood as $D^+ \to W^+ \to K^+K^-K^+$. In this topology, the decay amplitude is driven by an axial weak current and may be written as $\mathscr{A} = \langle (KKK)^+|A_\mu|0\rangle\langle 0|A_\mu|D^+\rangle$, where the latter is associated with the matrix element

$$\langle 0|A^\mu|D^+(P)\rangle = -i\sqrt{2}F_D P^\mu \;, \tag{1.1}$$

with F_D being a constant and $P = (p_1+p_2+p_3)$. The matrix elements $\langle (KKK)^+|A_\mu|0\rangle$, is what we called Multi-Meson Model, or Triple-M for short. The Triple-M amplitude includes two-body final state interactions, within the K-matrix approximation, and contains three components: $(KKK)^+$ nonresonant, $f_0(980)K^+$ and ϕK^+, all derived from a chiral effective theory and dressed with coupled channels where appropriate.

The universal character of this matrix elements allow one to easily extend this formalism to other heavy meson decays into three kaons, together with other topologies that must be considered.

2. Model

The Triple-M concentrates on effects associated with the matrix element $\langle K^- K^+ K^+|A^\mu|0\rangle$, which is especially relevant for the decay $D^+ \to K^- K^+ K^+$ if we assume the decay to be dominated by the process shown in Fig. 1. Our main target with this first model is identify the simplest possi-

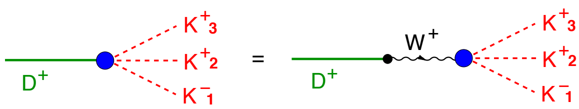

Figure 1: The decay $D^+ \to K^- K^+ K^+$ (left) is assumed to proceed thought quark-annihilation topology in the steps $D^+ \to W^+$ and $W^+ \to K^- K^+ K^+$ (right).

ble structures which could be instrumental to empirical data analyses. Therefore, one consider only resonances with isospin zero, and the tree-level matrix element $\langle K^-(p_1)K^-(p_2)K^+(p_3)|A^\mu|0\rangle$ is given in Fig. 2. The top line displays the leading order (LO) contact terms, whereas NLO corrections are given within brackets. All matrix elements are calculated by means of chiral effective Lagrangians including resonances, developed by Ecker, Gasser, Pich and De Rafael [4], where all the formalism needed can be found.

PoS(CHARM2016)086

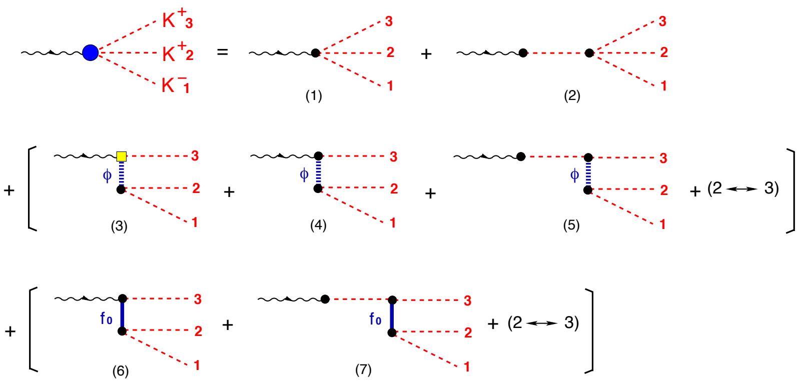

Figure 2: Tree-level structure for the $A^\mu \to K^- K^+ K^+$ matrix element: the top line is LO and terms within brackets, which involve ϕ and f_0 intermediate states, are NLO; there are two different forms for the $WK\phi$ coupling, indicated by a yellow box and a black dot.

The treatment of the ϕ-meson includes the $\pi\pi\pi$ intermediate channel and one consider the propagator dressed by $\rho\pi$ intermediate states, denoted by $[D_\phi^{\pi\rho}]^{-1}$. For the scalar $f_0(980)$, given that the $SU(3)$ structure is not clear, we allow it to be either a singlet or a member of an octet. In the sequence these two possibilities are labelled by $\gamma_0 = 8$ and $\gamma_8 = 1$, respectively.

Adding all the contributions together, the tree amplitude can be written as:

$$T_{tree} = C \left\{ \left[M_D^2 + M_K^2 - m_{23}^2\right] - \left[\sin^2\theta \, \frac{3\,G_V^2}{4F_K^2}\right] \left[\frac{m_{12}^2}{D_\phi^{\pi\rho}(m_{12}^2)} \left(m_{13}^2 - m_{23}^2\right) + 2 \leftrightarrow 3\right] \right. \tag{2.1}$$

$$\left. - \left[\frac{\gamma_n}{6\,F_K^2}\right] \left[\frac{1}{m_{12}^2 - m_{f_0}^2} \left[c_d \left(m_{12}^2 - M_K^2\right) - (c_d - 2\,c_m)\,M_D^2\right] \left[c_d\, m_{12}^2 - 2\,(c_d - c_m)\,M_K^2\right] + 2 \leftrightarrow 3\right] \right\} .$$

where θ is the $\omega - \phi$ mixing angle; G_V and c_d, c_m are the vector and scalar coupling constants defined in Ref. [4]; and C is the constant

$$C = \left[\frac{G_F}{\sqrt{2}} \sin^2\theta_C\right] \frac{2F_D}{F_K} \, \frac{M_K^2}{M_D^2 - M_K^2} \, ; \tag{2.2}$$

where θ_C is the Cabibbo angle and F_D, F_K the vacuum expectation value for D and kaon mesons.

2.1 Final State Interactions

The full decay amplitude is obtained by including final state interactions in the processes of Fig. 2. Here we consider only two-body rescattering process with $I = 0$ and angular momentum $J = 0, 1$. In diagrams (1a) and (2a) we consider the $\bar{K}K$ rescattering associated only with the ϕ channel. The second class of FSI accounts for production amplitudes which endow the ϕ and f_0 propagators with their full widths. In practice, the ϕ production amplitude accounts for independent widths for K^-K^+, $\bar{K}^0K^0$, and $\rho\pi$ decay modes, whereas $f_0(980)$ production amplitude accounts for $\bar{K}K$, $\pi\pi$ and $\eta\eta$. We work within the K-matrix approximation and, therefore, skip loop contributions from off-shell states.

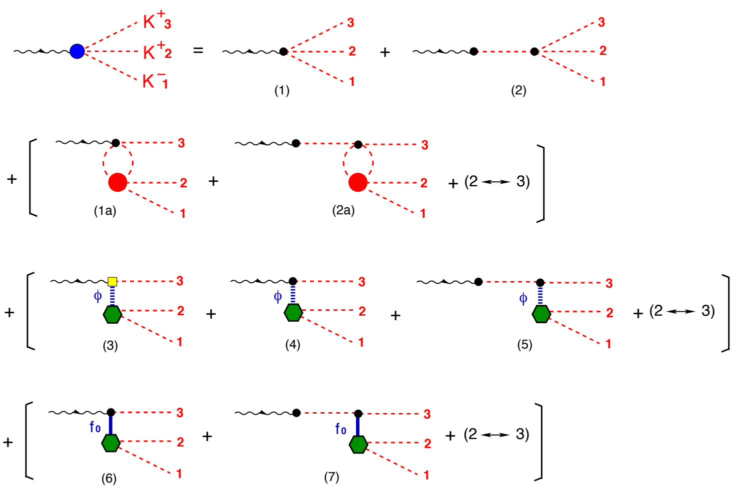

Figure 3: Dynamical structure of the $A^\mu \to K^-K^+K^+$ matrix element, including final state interactions: the top line, diagrams (1) and (2), is the LO nonresonant contribution, diagrams (1a) and (2a) include a KK rescattering, indicated by the red blob; whereas diagrams (3-7) describe ϕ and f_0 contributions with their full widths; there are two different forms for the $WK\phi$ coupling, indicated by a yellow box and a black dot.

The nonresonant contribution is determined at tree-level. Dynamically, it is a proper three-body amplitude, a direct consequence of chiral symmetry, which predicts multi-meson topologies.

$$T_{NR} = C\,\left[M_D^2 + M_K^2 - m_{23}^2\right]\,, \tag{2.3}$$

$$= C\,\left[(m_{12}^2 - M_K^2) + 2 \leftrightarrow 3\right]\,. \tag{2.4}$$

The second form makes it clear that T_{NR} contains just S-waves.

Our studies for the ϕ channel shows that contact interactions have little numerical relevance in the phase-space accessible to the $D^+ \to K^+K^-K^+$ decay. Also, one find out that the relevance of dressing the ϕ propagator with $\pi\rho$, $D_\phi^{\pi\rho}$, is small and, for the sake of simplicity, both can be safely removed from the model. Then, the final contribution for this channel became:

$$T_\phi = -C\left[\sin^2\theta\,\frac{3\,G_V^2}{4F_K^2}\right]\left[m_{12}^2\,\frac{[m_{13}^2 - m_{23}^2]}{D_\phi(m_{12}^2)} + 2 \leftrightarrow 3\right]\,, \tag{2.5}$$

$$D_\phi(s) = s - m_\phi^2 + i\,m_\phi\,\Gamma_\phi(s)\,, \tag{2.6}$$

$$m_\phi\,\Gamma_\phi(s) = \sqrt{s}\left[\Gamma_{KK}\,\frac{(Q_c^3 + Q_n^3)}{(\tilde{Q}_c^3 + \tilde{Q}_n^3)} + \Gamma_{\pi\rho}\,\frac{s}{m_\phi^2}\,\frac{Q_{\pi\rho}^3}{\tilde{Q}_{\pi\rho}^3}\right]\,. \tag{2.7}$$

where $Q_{\pi\rho} = \frac{1}{2}\sqrt{s - 2\,(M_\pi^2 + m_\rho^2) + (M_\pi^2 - m_\rho^2)^2/s}$, $Q_c = \frac{1}{2}\sqrt{s - 4M_{K^+}^2}$, $Q_n = \frac{1}{2}\sqrt{s - 4M_{K^0}^2}$ and $\tilde{Q} = Q(s = m_\phi^2)$. There are no free parameters in T_ϕ. All the couplings and constants are extracted from previous references [4, 5, 8].

The most interesting feature in ϕ channel is the factor m_{12}^2 in the numerator of eq.(2.5), and the $\sqrt{s}$ outside the bracket in eq.(2.7), which are signatures of resonance couplings in chiral perturbation theory. Moreover, these signatures are the main difference if one compare the ϕ structure with the standard Breit-Wigner and the modulus of the latter falls faster than that of the Triple-M at high values of s.

In the f_0 channel, one found that the contribution for $\eta\eta$ is negligible and can be excluded. The final amplitude for the singlet($n = 0$) and the octet ($n = 8$) is:

$$T_{f_0} = -C\left[\frac{\gamma_n}{6F_K}\right]\left[\left[c_d\left(m_{12}^2 - M_K^2\right) - (c_d - 2c_m)M_D^2\right]\frac{G_K(m_{12}^2)}{D_n(m_{12}^2)} + 2 \leftrightarrow 3\right], \tag{2.8}$$

$$D_n(s) = s - m_{f_0}^2 + i m_{f_0}\Gamma_n(s), \tag{2.9}$$

$$m_{f_0}\Gamma_0 = \frac{G_\pi^2}{4\pi F_\pi^2}\frac{Q_{\pi\pi}}{\sqrt{s}} + \frac{G_K^2}{3\pi F_K^2}\frac{Q_{KK}}{\sqrt{s}}, \quad m_{f_0}\Gamma_8 = \frac{G_\pi^2}{8\pi F_\pi^2}\frac{Q_{\pi\pi}}{\sqrt{s}} + \frac{G_K^2}{24\pi F_K^2}\frac{Q_{KK}}{\sqrt{s}};$$

$$G_P(s) = \frac{\left[c_d\, s - 2\,(c_d - c_m)\,M_P^2\right]}{F_P}, \quad Q_{PP}(s) = \frac{1}{2}\sqrt{s - 4M_P^2},$$

for $P = \pi, K$. The mesons decay constants are extracted from literature [3, 8], and the values of c_d and c_m are to be determined by fits to data. In ref.[4] they estimates c_d, $c_m = 0.032, 0.042\,\mathrm{GeV}$ which provides an educated guess of departure.

The most striking feature of the f_0 in the Triple-M is the presence of s-dependent couplings, predicted by chiral perturbation theory. This means that the amplitude T_{f_0} is somewhat flexible, since it depends on two free coupling parameters: c_d and c_m. We obtained different values for c_d and c_m by fixing our amplitude at the energy of the resonance with the standard Flatté function [6], using parameters obtained by BES II [$g_{\pi\pi} = 0.165\,\mathrm{GeV}$ and $g_{KK} = 0.695\,\mathrm{GeV}$] [7]. In Fig.4, we compare the result of $T_{f_0}/[-C]$ using this new values together with those obtained by ChPT[4]. Indeed, our studies showed that the final line shape for the amplitude is rather sensible for the values of those constants.

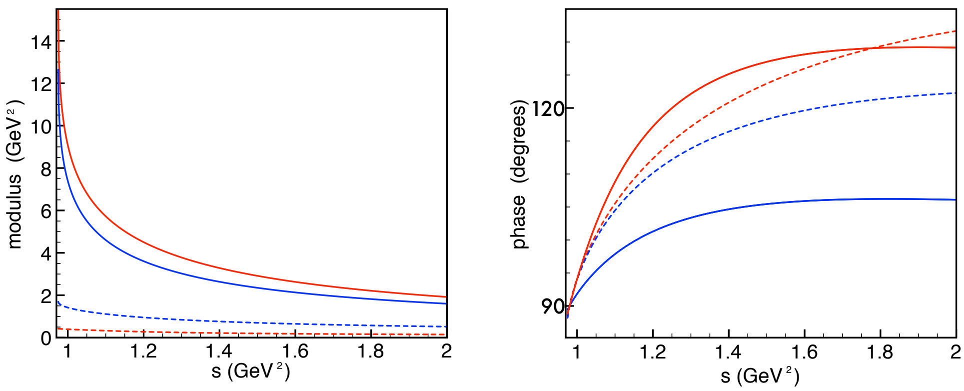

Figure 4: Moduli(left) and phases(right) of the ratio $T_{f_0}/[-C]$ for the singlet (blue curves) and octet (red curves) cases, based on the values of c_d and c_m extracted from Flatte function[7] (continuous curves) and on those given in Ref. [4] (dashed curves).

3. Monte Carlo simulation

The full Triple-M amplitude include a nonresonant contribution (NR), supplemented by ϕ and f_0 resonant terms, and is formally written as

$$T_{MMM} = T_{NR} + T_\phi + T_{f_0} \,. \tag{3.1}$$

We generate a toy monte-carlo with parameters discussed above and the resulting Dalitz Plot is presented in Fig. 5.

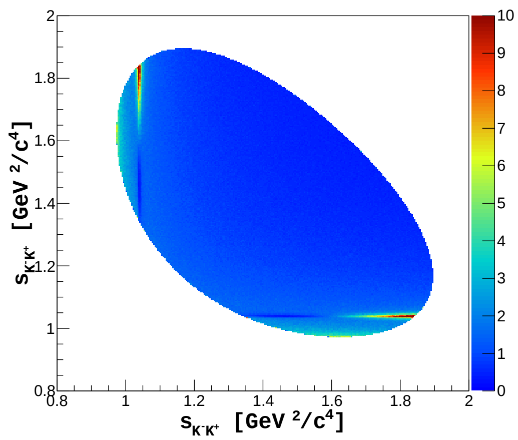

Figure 5: Dalitz plot with MC simulation of the full Triple-M amplitude. The f_0 is assumed to be a singlet state. The hypothesis of the f_0 being an octet state yields a nearly identical distribution.

One interesting feature is the distribution of events in the ϕ region. One of the lobes is depleted with respect to the other, resulting in a peak and a dip. This is caused by the interference between the ϕ and the f_0 components of the Triple-M amplitude. In this region the strong phases are rapidly changing. Interestingly, the $D^+ \to K^+K^-K^+$ LHCb data presented by A. C. dos Reis at this conference[1] shows the same kind of interference. It also, very important to emphasize that the resulting distribution of events is very sensitive to the details of the parametrization of these two components.

4. Final remarks

We presented a first version of the Multi-Meson-Model, or Triple-M, applied to $D^+ \to K^+K^-K^+$ decay. Details of the calculations can be found in Ref. [1] and a more complete model is on the way [9]. The most important feature of the Triple-M amplitude is that the relative contribution and phase of each component is fixed by theory, an important difference from the isobar model. The resonant contributions involve expressions which are very different from the A_k used in the isobar model amplitude $A = \sum c_k A_k$ [10], although these expressions yield a similar line shape. In the Triple-M, the free coefficients c_k are absent, because the intensity of each resonance is predicted

[1] A. C. dos Reis contribution to these proceedings

by the underlying dynamics. In particular, the ϕ contribution is completely fixed, for its intensity is related directly with the decay width into $\bar{K}K$. The case of the f_0 is different because one does not have precise values for its mass and couplings. Therefore, the three parameters in the amplitude, namely m_{f_0}, c_d, and c_m, are left to be determined by fits to data.

References

[1] R.T. Aoude, P.C. Magalhães, A. C. dos Reis and M.R. Robilotta, [arXiv:1604.02904]

[2] L-L. Chau, Phys. Rep. **95**, 1 (1983).

[3] J. Gasser and H. Leutwyler, Ann. Phys. **158**, 142 (1984); Nucl. Phys. B**250**, 465 (1985).

[4] G. Ecker, J. Gasser, A. Pich and E. De Rafael, Nucl. Phys. B **321**, 311 (1989).

[5] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016).

[6] S.M. Flatté, Phys. Lett. B **63** , 224 (1976).

[7] M. Ablikim *et al.* , BES Collaboration, Phys. Lett B 607, 243 (2005).

[8] M. Jamin, J.A. Oller and A. Pich, Nucl. Phys. B **587**, 331 (2000).

[9] R.T. Aoude, P.C. Magalhães, A. C. dos Reis and M.R. Robilotta, work in progress.

[10] Particle Data Group, K. Nakamura et al., Review of Particle Physics, Dalitz plot analysis formalism, J. Phys. G 37 (2010) 075021